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RAND-McNALLY

PRIMARY ARITHMETIC

PRINCIPLES AND PROCESSES DERIVED
BY INDUCTION.

FOR PUPILS OF THE FOUR LOWEST GRADES.

From things to thoughts.

BY

EDWIN C. HEWETT, LL. D.,
EX-PRESIDENT OF THE ILLINOIS STATE NORMAL UNIVERSITY.

CHICAGO AND NEW YORK:
RAND, McNALLY & COMPANY.

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INTRODUCTION.

THE aim of this book is to lead the beginner to a knowledge of the principles of numbers in such a way that he shall comprehend clearly every step. It is a law of mind that the first notions in any department of thought are acquired through the use of the senses, *through sense perception.* Hence, we have introduced, at every step, the oral lessons dealing with a variety of objects. But it is of great importance to remember that objects should be used no longer than they serve their purpose; as soon as the child has grasped the *idea* firmly, the use of objects should be discontinued. The teacher must determine, by careful observation, when this stage is reached. Very likely many children will need less of this oral work than is here given; others may need even more. The teacher is not asked to follow these exercises blindly and mechanically; they are intended to be suggestive merely.

The teacher should remember that the oral exercise is to lead up to the matter in the lesson "For Study"; hence, that exercise should always precede the study lesson. After the child has mastered the lesson "For Study," it is expected that he will be called upon to recite it, or to be examined upon it, in the usual manner of conducting class recitations. The oral lesson is to prepare him for the study; the recitation is to determine the success of his study.

This book calls for the employment of an unusual amount of time in the early steps. It does not demand a great deal of memorizing, but what it does ask for should be *perfect.* It fully recognizes the truth that slow but *sure* progress at the beginning is certain to result in a great saving of time in the end.

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TO THE TEACHER.

IN order that the teacher may use this book in the most successful way, it is necessary that she should give careful heed to the following suggestions:

The book is designed for young children; it is supposed that they have no knowledge of numbers except such as they may have picked up unconsciously in their plays, and in their home life. They will often be found able to count — that is, to say the names of the numbers in order — as far as to twenty or farther.

THE CHILD'S CONDITION.— It is the teacher's first business to ascertain with respect to every child in the class, just how far his present knowledge extends, so that she may *waste no time in attempting to teach him what he knows already;* nor, on the other hand, attempt to teach him what is so far in advance of his present knowledge that he will be incapable of understanding it.

The first lessons in this book are constructed on the supposition that the child can read with some fluency; that he can write and draw to some extent. If the teacher attempts to use the book with children who are not up to this standard, she will need to aid them accordingly.

HOW TO USE THE BOOK.—With children in the primary grades, most of the successful teaching must be *oral.* To help the teacher in this work, the oral exercises preceding each lesson are specially prepared. These exercises and questions are intended to be *suggestive* merely; the good teacher will devise exercises for her own use, either additional to those given, or in place of some of them. Enough exercises of this kind should be used to accomplish their purpose, *but no more.* When the child knows instantly and without effort that five and two are seven, with the facts implied in that knowledge, it is a very great mistake to drill him any longer on exercises which aim to teach those facts.

The child's lessons are intended chiefly to furnish him seat work; the oral exercises are intended to help the teacher in preparing the child to do his seat work intelligently. When the pupil's seat work is completed, the teacher should not fail to examine it carefully, noting both its *accuracy* and its *neatness.*

SOME FUNDAMENTAL PRINCIPLES.— *Use objects.* In Arithmetic, as in everything else, the child gets his *first conceptions through the use of his senses.* Hence, all his first operations with numbers should be performed with objects. To insure variety and to promote interest, the teacher should be supplied with an abundance of objects, such as buttons, spools, kernels of corn, beans, nails, etc. For a good reason, which will appear in the course of our work, splints of wood, something like jackstraws or toothpicks, are recommended for most frequent use. Splints especially prepared for this purpose are for sale by all the principal dealers in supplies for primary schools; but the teacher can readily prepare them for herself, or get some older pupil to do so.

In every case the child should use splints, or counters of some kind, till he has mastered the general truth to be learned, but *no longer.* It is as great an error to continue the use of objects too long as it is to attempt to teach numbers without using objects at all, which was formerly the custom. If at any time, however, it is found that a pupil has failed to learn what he was supposed to have learned from the use of his counters, turn him back to their use temporarily, and thus correct his deficiency.

Do not confound figures with numbers. It is believed that failing to distinguish between numbers and the figures that represent them is the source of a vast deal of confusion in pupils' minds. We do not "add figures," "multiply one figure by another," find "how many times one figure is contained in another," etc. To keep the matter clear from the start, many questions are suggested in these lessons similar to the following: Where is the number? Where is the figure that "stands for it"? Where is the number that this figure "stands for"? etc.

How many? What? Essentially, number is *ratio.* The number *seven,* used abstractly, signifies merely the ratio of a quantity of units of any kind to one unit of the same kind by which the quantity is measured. But no one can have a clear thought of the measured quantity till he knows clearly what the measuring unit is. Now this is so important that we deem it essential that it should be brought to the pupil's consciousness even from the start. Hence the frequency of the questions, How many? What? or What kind? in the suggested exercises.

PRIMARY ARITHMETIC.

PART I.

LESSON 1.

NOTE.—The following oral exercises prepare pupils for written work during the study hour. Be sure that many of the exercises call for *action* on the part of the child. (The exercises with the class should precede the pupil's seat-work.)

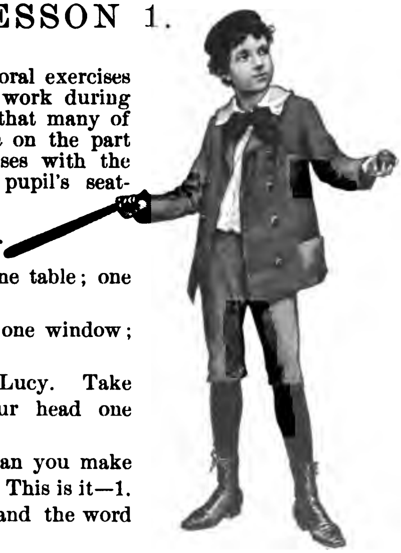
ORAL EXERCISE.

Show me one book; one table; one stick of crayon.

Point toward one boy; one window; one door; one tree.

Bring me one pencil, Lucy. Take one step, Fred. Nod your head one time, Charley.

Can you spell *one?* Can you make the figure that means one? This is it—1. (Teacher makes the figure and the word on the board.)



1 One

FOR STUDY.

See this boy. How many boys are there? How many heads has he? How many noses? How many mouths? What has he in his hands? How many bats do you see?

Make the figure 1, and write the word *one*, carefully on paper at your seat.

LESSON 2.

ORAL EXERCISE.

How many hands have you?

Show me as many fingers as you have hands; as many splints.

Show me two shells; two spools; two table-legs.

(Let the teacher substitute the names of pupils for the names in the book.)

Harry, bring me two books. Two pencils, Mary. Two sticks of crayon, Ethel. Two buttons, Albert.

Each of you may take one block in your right hand. Take one block in your left hand. Now put these blocks together. How many blocks in all?

One block and one block taken together are how many blocks?

Hattie may bring me one pencil. Elmer may bring me one pencil. I will put these pencils together. How many pencils together have I?

One pencil and one pencil are how many pencils?

Mabel had one doll, and her aunt gave her one more. How many dolls had Mabel then?

Fred may tell the story of Mabel and her dolls.

Fred: "Mabel had one doll. Her aunt gave her one more. She then had two dolls."

George wanted a top that would cost him two cents. He had one cent. How many more cents must he get?

Each of you may hold up two splints. Now you may lay one of them on the table. How many splints have you left?

George may tell the story.

George: "I had two splints and laid one splint on the table. I then had one splint left."

Two blocks less one block are how many blocks?

Maggie may make two marks on the board. Willie may erase two of them. How many marks are left?

Two marks less two marks leave how many marks?

Irene may tell the story.

Irene: "Maggie made," etc.

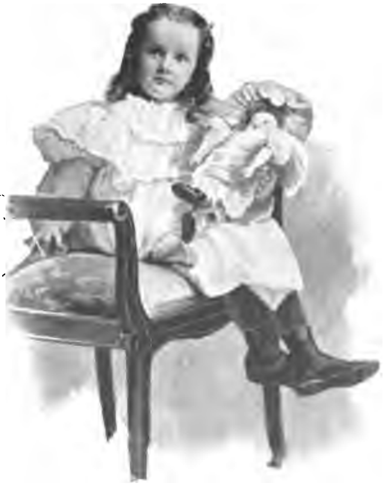
(Incidentally call the pupil's attention to the changes in words, when more than one is meant; that is, to the formation of plurals.)

Answer as quickly as you can, using only one word:

One and one are how many?
Two less one are how many?
Two less two are how many?
One less one are how many?
What is one half of two?
How many ones are there in two?
How many are two times one?
How many twos are there in two?
Who can write the word two?

The *figure* that means *two* is at the left of the picture of the little girl. The *word two* is at the right of the picture.

NOTE.—These exercises will suggest to the teacher the variety that can be introduced. They may be extended as much as is necessary, or any may be omitted if not needed. Suit the work to the child's condition. All the work should proceed as promptly as it is possible for the pupils to follow it.



2 **Two**

FOR STUDY.

See this girl.

How many girls are there?

How many eyes has she?

How many feet?

How many hands has the doll?

How many arms?

How many feet has the doll?

How many feet have you?

Make one short line on your slate. Now make one more beside it. How many lines taken together have you made?

One line and one line are how many lines?

Make two nice dots on your slate. Erase one of them. How many dots are left?

Two dots less one dot leave how many dots?

There was one mouse in a trap and one mouse more ran into the trap. How many mice were then in the trap?

One mouse and one mouse are how many mice?

John had one marble. He found enough more to make two marbles. How many did he find?

Harry started home with two books. He lost one on the way. How many books did he have when he reached home?

Two apples less two apples leave how many apples?

Copy and fill blank spaces with the right words:

One fox and one fox are —— foxes.
Two apples less —— apple leave one apple.
One cat and —— cat are two cats.
Two pigs less —— pigs leave no pigs.

Make the figure 2, and write the word *two*, carefully on your slates.

LESSON 3.

ORAL EXERCISE.

Each of you may show me as many splints as there are cows in the picture on page 14; as many fingers; as many buttons.

How many buttons did you show me?

Show me three shells; three grains of corn; three fingers.

Ralph, point toward three windows. Point toward three girls, Minnie. All may take three steps forward.

NOTE.—Let the teacher show three objects, as three sticks of crayon, three nails, three clothespins, etc., and first ask, "How many?" then ask, "What kind?" From the first, pupils should recognize these two facts about every number, viz.: How many units? What kind of units?

Hold up two fingers on your right hand. Now hold up one finger on your left hand. Put all these fingers together. How many fingers are you holding up?

Two fingers and one finger are how many fingers?

One finger and two fingers are how many fingers?

Willie may make one mark on the board. Lucy may make two more marks beside the one Willie made. How many marks have Willie and Lucy together made?

John may tell the story.

One mark and two marks are how many marks?

Two marks and one mark are how many marks?

(The teacher should be sure that the pupils see clearly that the sum of two and one is the same as the sum of one and two.)

You may all take two splints in one hand and one splint in the other. (The teacher will do the same.)

I am going to tell a story about two and one, and then I will ask some of you to tell a story about two and one.

"My splints are apples. Frances has two apples (holding up two splints) and her brother gave her one more (bringing up one splint and putting it with the two splints). She then had three."

Richard may tell me a story about two and one.

(Richard might say · "My splints are marbles. Frank had two marbles (holding up two splints) and he found one more marble (putting the one splint with the two splints); he then had three marbles.")

Flora may tell a story about one and two.

(Flora might say: "My splints are kittens. One kitten was playing on a rug and two more kittens came to play with it. Then there were three kittens on the rug.")

(The teacher should see that the child suits his actions to his words when he tells stories like these.)

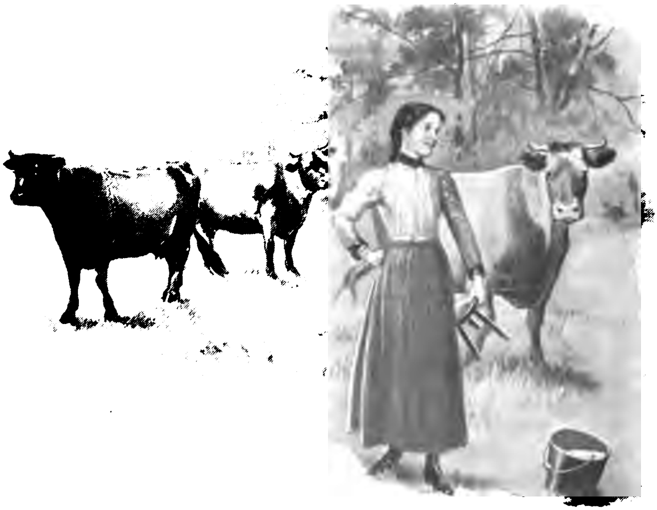
Albert had one cent. His father gave him two cents more. How many cents had he then?

A farmer needs three horses to work his plow. He has only two horses. How many horses must he buy?

Answer quickly with one word:

One and one are how many?
Two and one are how many?
One and two are how many?
What is one half of two? Show me.
How many ones in two?
How many ones in three? Show me.
What is one third of three splints? Show me.
One is what part of three?
How many are three times one?

The figure that means three is at the left-hand side of the picture of the cows, and the word *three* is at the right-hand side.



3 FOR STUDY. **Three**

What do you see in this picture?
How many cows are there?
How many legs has the stool?
Make two dots on your slate. Make one more

dot beside the two dots. How many dots have you made?

Two dots and one dot are how many dots?

Two cows and one cow are how many cows?

Make one short mark on your slate. Make two more marks beneath the first one. How many marks have you made?

One mark and two marks are how many marks?

Two marks and one mark are how many marks?

One mark and one mark and one mark are how many marks? Show it on your slate.

Three times one mark are how many marks?

One book and two books are how many books?

Two boys and one boy are how many boys?

One girl and one girl and one girl are how many girls?

Fred has two cents. How many cents must he get to have three cents?

Hattie had one plum and her sister gave her two more plums. How many plums had she then?

Copy and fill blank spaces with the right words:

Two mice and —— mouse are three mice.
One peach and —— peaches are three peaches.
Two wagons and one wagon are —— wagons.
One half of two oranges is —— orange.
There are —— ones in two.
There are —— ones in three.
Three times one are ——.

Make the figure 3 and the word *three* each three times.

LESSON 4.

ORAL EXERCISE.

Two splints and one splint are how many splints?

One splint and two splints are how many splints?

Show me the number of splints which these figures (making them on the board) stand for: 3, 2, 1, 3, etc.

You may all pass to the board.

Make the figure which tells me the number of spools I show you. (Here the teacher holds up three spools, one spool, two spools, three spools, etc.)

Where are the spools?

Where are the figures which stand for them?

Each of you may take three splints. Put one of them behind you. How many are left in sight?

Three splints less one splint are how many splints?

Fannie may make three circles on the board.

Grant may erase two of them.

Three circles less two circles are how many circles?

Each of you may hold up three buttons. Lay three of them on the table. How many buttons have you left?

Three buttons less three buttons are how many buttons?

I will tell you a story about three less two:

"My splints are birds. There were three birds sitting in a tree (holding up three splints). Two of them flew away (taking two splints away from the three splints and holding them down by the side). There was only one bird left in the tree."

Julia may tell a story about three less one.

(Julia might say: "My splints are roses," etc.)

Take three splints. How many ones in three? Show it with your splints. Show me one third of three splints.

One is what part of three?

Three times one are how many?

How many twos in three? How many are left over? Show it with splints.

Harry had two cakes. He ate one. How many had he left?

Jennie had three cups. She broke two. How many were left?

A farmer had three pigs in a pen. When he went to feed them, he found only one pig. How many pigs were missing?

Quick answers:

Three less one are how many?
Three less two are how many?
Three less three are how many?
Two and one are how many?
One and two are how many?
One and one are how many?
One and one and one are how many?
Three times one are how many?
What is one half of two?
What is one third of three?
One is what part of three?
One is what part of two?

FOR STUDY.

Make three crosses on your slate. Erase one of them. How many crosses have you left?

Three crosses less one cross leave how many crosses?

Make three more crosses on your slate. Erase two of them. How many crosses are left?

Three crosses less two crosses are how many crosses?

Once more make three crosses on your slate. Erase three of them. How many are left?

Three crosses less three crosses are how many crosses?

John brought three marbles to school and lost one of them. How many marbles had he left?

Copy and fill blank spaces:

Three chickens less —— chicken are two chickens.
One half of two pears is —— pear.
One third of three turkeys is —— turkey.
One rose and —— roses are three roses.

LESSON 5.

ORAL EXERCISE.

(Let the teacher be certain that the pupils have done their written work correctly and neatly.)

How many fingers have you on one hand, not counting the thumb?

Each of you may show me as many splints as you have fingers on one hand.

How many did you show me? What were they?

How many of each have I? (Showing four spools, four pencils, four books, etc.) What kind are they?

How many legs has my table?

One and how many are four?

Edward may count off four boys, beginning with Frank. How many did he count off? What are they?

Each of you may take three splints in your right hand and one splint in your left hand. Put these splints together into one hand. How many all together? What are they?

Three splints and one splint are how many splints?

One splint and three splints are how many splints?

How many blocks (holding up one block) have I in my right hand?

I will give this one block to Nancy. How many more blocks must Nancy have to make four blocks?

One block and three blocks are how many blocks?

Fred may tell a story about three and one, using splints.

Lucy may tell a story about one and three.

Robert may tell a story about one and one and one.

Quick answers:

Three and one are how many?

One and three?

Three less one?

One and one and one are how many?

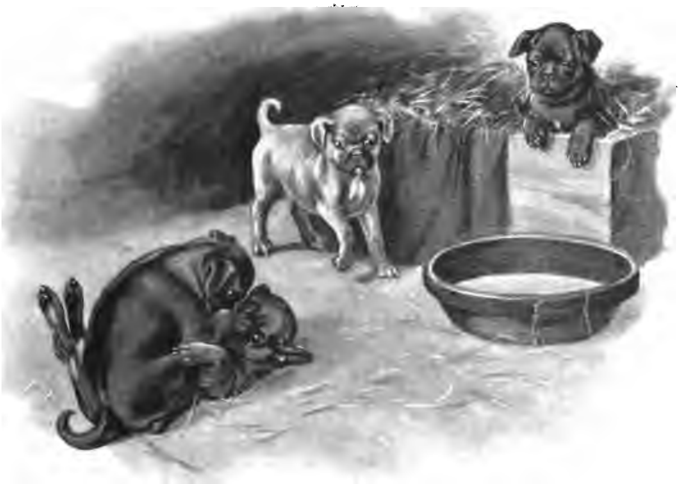
Three times one are how many?

One and one and one and one are how many?

Four times one are how many?

How many letters in the word *four?*

See the figure 4 and the word *four* by the picture of the pups.



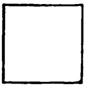
4 FOR STUDY. Four

What nice pups! How many pups are there?

How many feet has one pup?

How many fingers have you on one hand, not counting your thumb?

How many corners has your slate?



How many sides has this square?

How many walls has your school-room?

How many corners has the ceiling?

How many feet has your desk?

How many letters in the word *four?*

Make one square on your slate. Just beneath this one square make three squares. How many squares have you made all together?

One square and three squares are how many squares?

Three squares and one square are how many squares?

Three tops and one top are how many tops?

One pencil and three pencils are how many pencils?

Copy and fill blank spaces:

Three kites and one kite are —— kites.
One sled and three sleds are —— sleds.
Three cakes less one cake are —— cakes.
Three cakes less —— cakes are one cake.

Write the word *four*, and make the figure 4, nicely on paper, each four times.

LESSON 6.

ORAL EXERCISE.

How many am I holding up? (Holding up four.) What are they? How many? What? (Spools.) How many? What? (Nails.) (Ask both questions every time.)

Show me the number of splints these figures (making them one at a time, on the board) stand for: 4, 3, 2, 1, 3, 4, 2.

You may pass to the board and make the chalk tell me the number I show you. (The teacher will hold up four clothespins, two pencils, etc., and the pupils will make the figure which tells the number. After the figure is made each time, she should ask such questions as these: "How many have I?" "What kind?" "Where are the two pencils?" "Where is the figure 2?" "Can you show me the figure 3?" "Can you show what it stands for?" etc.)

Take four buttons in your right hand. Put one of them behind you. How many buttons are left in your right hand?

Four buttons less one button are how many buttons?

Harold may bring me four sticks of crayon. Mabel may take three of them away. How many sticks of crayon have I left?

Jessie, tell the story.

Three shells and how many more will make four shells? Show it with splints.

Quick answers:

Three and one are how many?
Four less one?
Two and one?
One and three?
Three less two?
Four less three?
One and one and one and one are how many?
Four times one are how many?
Three less three are how many?

FOR STUDY.

Make four squares together on your slate. Erase one of them. How many squares are left?

Four squares less one square are how many squares?

Make four dots on your slate. Erase three of them. How many dots are left?

Four dots less three dots are how many dots?

Emma needs four spools of thread. She has only one spool. How many spools must she buy?

Fred wanted a top for which he would have to pay four cents. He had three cents. His father gave him enough more cents to buy the top. How many cents did his father give him?

Copy and fill blank spaces with the right figures:

3 lemons and 1 lemon are —— lemons.
4 mice less 3 mice are —— mouse.
1 hat and —— hats are 4 hats.
4 balls less —— balls are 1 ball.

LESSON 7.

ORAL EXERCISE.

Each of you may hold up two fingers. Now hold up two more fingers. How many are you holding up all together? What?

Two fingers and two fingers are how many fingers?

Each of you may take four splints. Lay two of them on the table. How many splints have you left?

Jennie may tell the story.

Four boys less two boys are how many boys?

Mary may place four girls in a row. Now she may stand in the middle of the row.

How many girls are at her right hand?

How many girls are at her left hand?

How many twos in four?

What is one half of four girls?

Two girls and two girls are how many girls?

Two times two girls are how many girls?

Each of you may take four splints. Separate them into two equal parts. How many splints are there in each part?

What is one half of four splints?

Each of you may take four splints. Separate them into four equal parts. How many splints are there in each part?

One splint is what part of four splints?

What is one fourth of four boys? Show me.

Alice may take four spools and give one spool to as many girls as she can.

How many girls got one spool?

How many ones are there in four?

Four times one are how many?

What is one fourth of four spools?

What part of four spools did one girl get?

Quick answers:

Four less two are how many?

Four less three are how many?

Four less one are how many?

Two and two are how many?

How many twos are there in four?

What is one half of four?

George had four cents. He paid two cents for a top. How many cents had he left?

Ella bought four spools of thread. One half of them were white. How many spools of white thread did she buy?

FOR STUDY.

Make two squares on your slate. Make two more squares just beneath the first two squares. How many squares have you made all together?

Two squares and two squares are how many squares?

Tom had two cents and his mother gave him two more cents. How many cents did he have then?

You have four squares on your slate. Erase two of them. How many squares have you left?

Four squares less two squares are how many squares?

Make four dots in a row on your slate. Draw a line between the two dots in the middle. How many dots are on the left side of the line? How many are on the right side of the line?

How many twos are there in four?

Two twos are how many?

Two times two are how many?

What is one half of four apples?

Two is what part of four?

How many pairs of shoes are there in four shoes?

How many ones are there in four?

How many tops at four cents apiece can you buy for four cents?

How many fours are there in four?

Copy and fill blank spaces:

4 kittens less 2 kittens are —— kittens.
3 desks and —— desk are 4 desks.
4 pears less —— pears are no pears.
2 fingers and —— fingers are 4 fingers.
One half of 4 horses is —— horses.
Two times 2 boys are —— boys.
Four times 1 girl are —— girls.

LESSON 8.

ORAL EXERCISE.

NOTE.—Show five objects (pencils, or clothespins, or roses), and ask the pupils to show as many splints, books, crayons, buttons, marks, etc.

Ask each time: How many did you show me? What?

You may show me five nails, five shells, five beans, etc.

Fred may take five steps.

Hattie may make five crosses on the board.

You may all raise and lower your left hand five times.

Each of you may hold up four fingers on one hand and one finger on the other. Put these fingers together. How many fingers all together?

Four fingers and one finger are how many fingers?

One finger and four fingers?

Robert may tell me a story about one and four, illustrating it with splints.

Emma may put five spools into my hand. Arthur may take one of them out. How many spools are left in my hand?

Five spools less one spool are how many spools?

Edgar may place five boys in a row.

Now he may count out four of them to take their places again in the class. How many boys are left?

Five boys less four boys are how many boys?

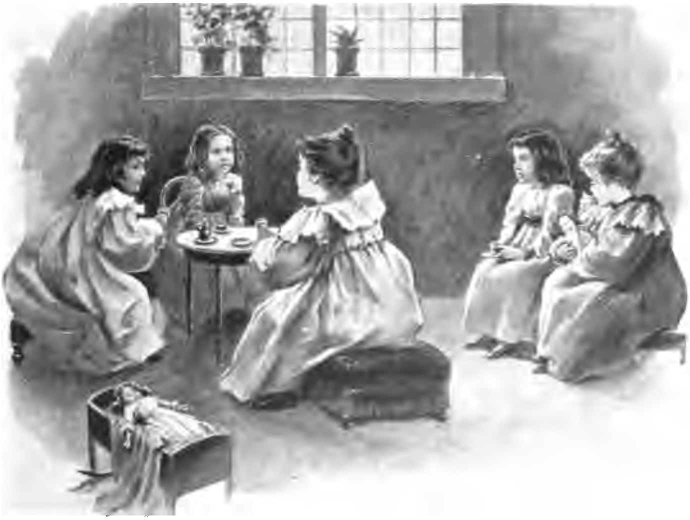
Helen may take five nails and tell a story about five less four.

Quick answers:

Four and one are how many?
One and four?
One and three?
Five less one?
Five less four?
Two and two?
One half of four?

Laura bought a spool of thread for four cents. She gave the clerk a nickel. How many cents should the clerk give back?

How many wheels have a wagon and a wheelbarrow together?



5 FOR STUDY. Five

How many girls in this picture? What are they doing?

How many cups must there be so that each girl may have one?

How many saucers? How many spoons?

How many seats must there be if each girl has one?

Counting the thumb, how many fingers have you on one hand?

Make four short, straight lines on your slate. Now make one more line beside them. How many lines in all have you made on your slate?

Four lines and one line are how many lines?

One line and four lines are how many lines?

Make four squares on your slate. Now make one more square. How many squares have you made in all?

Four squares and one square are how many squares?

One square and four squares are how many squares?

Make five dots on your slate. Now erase one of them. How many dots are left?

Five dots less one dot are how many dots?

Make five crosses on your slate. Now erase four of them. How many crosses are left?

Five crosses less four crosses are how many crosses?

Five boys less one boy are how many boys?

Elmer has four marbles in his pocket and one in his hand. How many marbles has he?

There was one pig in a pen and the farmer put four more pigs into the pen. How many pigs were in the pen then?

Alice has four cents. How many cents must she get to have five cents?

Frank had one marble. He found enough more to make five marbles. How many marbles did he find?

Copy and fill blank spaces:

4 windows and 1 window are —— windows.
1 slate and 4 slates are —— slates.
5 books less 1 book are —— books.
5 girls less 4 girls are —— girl.
3 cows less —— cows are 1 cow.
One half of 4 pencils is —— pencils.

This is the figure that means five—5. Write this figure and the word *five* neatly on your slate five times.

LESSON 9.

ORAL EXERCISE.

Four cows and one cow are how many cows?

One peach and four peaches are how many peaches?

Take three splints in the right hand and two splints in the left hand. Put them all into one hand. How many splints have you all together?

Three splints and two splints are how many splints?

Two splints and three splints are how many splints?

How many nails must I put with three nails to have five nails? Show me.

Jessie may take three nails and two nails and tell a story about three and two.

Ellen may tell a story about two and three, illustrating with buttons.

Five girls less two girls are how many girls? Show me.

Take five nails. Put three of them into this box. How many nails have you left?

Five nails less three nails are how many nails?

Sadie may tell a story about five less two.

Dean may tell a story about five less three.

Lawrence paid three cents for a whistle. He gave the clerk a nickel. How many cents should he receive back?

There were three boys on a sled when it started down hill. When the sled reached the foot of the hill there were five boys on it. How many boys jumped on after the sled started?

Mary's doll has five dresses and Laura's doll has two dresses. How many more dresses has Mary's doll than Laura's?

Quick answers:

Five less two are how many?
Five less one are how many?
Three and two?
Five less three?
Four and one?
Five less four?
Two and three?
One and four?

The sign $+$ (making the sign on the board) means *and*, and the sign $=$ means *are*.

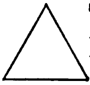
Read: $2+1=3$; $3+1=4$; $1+1=2$.

Copy and complete these (putting them on the board):

$4+1=$
$3+2=$
$1+3=$
$1+4=$
$3+1=$
$2+3=$
$2+2=$
$2+1=$
$1+2=$

Read them after they are completed.

FOR STUDY.



This is a triangle.

Make three triangles like it in a row on your slate. Make two more beneath the three. How many triangles have you made?

Three triangles and two triangles are how many triangles?

Make two dots on your slate. Make three more dots beneath them. How many dots have you made?

Two dots and three dots are how many dots?

Two trees and three trees are how many trees?

Three trees and two trees are how many trees?

Albert fed his pony two ears of corn and three ears of corn. How many ears of corn did he feed his pony in all?

Harry shot two ducks and George shot enough ducks so that both boys together had five ducks. How many ducks did George shoot?

Make five triangles on your slate. Erase two of them. How many are left?

Make five more triangles on your slate. Erase three of them. How many are left?

Five triangles less two triangles leave how many triangles?

Five triangles less three triangles leave how many triangles?

Five fingers less two fingers leave how many fingers?

Five fingers less three fingers leave how many fingers?

Albert caught five fish. He sold three of them. How many had he left?

Flora had five chickens. Two of them died. How many were left?

Willie has two marbles. I want to give him enough more so that he will have five marbles. How many must I give him?

Copy and fill blank spaces:

3 bears and 2 bears are —— bears.
2 cats and 3 cats are —— cats.
5 books less 2 books are —— books.
5 books less —— books are 2 books.
1 orange and —— oranges are 5 oranges.
5 boys less —— boys are 1 boy.
1 goose and —— geese are 4 geese.
3 mice less —— mice are no mice.

LESSON 10.

ORAL EXERCISE.

If Albert has five nuts and eats five nuts, how many nuts will he have left?

Roy may take five nails. Give two to as many boys as you can. How many boys got two nails? How many nails are left over?

Edith may tell the story.

Where is one of the twos? What kind is it? (Nails.) Where is the other two? What kind is it? Where is the one? What kind of a one is it?

How many twos are there in five? How many are left over?

Two fingers and two fingers and one finger are how many fingers? Show me.

Alice had two cents. Her father gave her two cents, and her mother gave her enough to make five cents. How many cents did her mother give her?

Lena may show it by giving splints to Alice, and tell the story as she does it.

Take five splints. Separate them into five equal parts. How many splints are there in each part? Who knows what part of five splints one splint is? Show me one fifth of five splints.

Louise may take five clothespins. Give one fifth of them to Nellie, and tell a story about one fifth of five.

Take five splints. Show me one fifth of them. Who can show me two fifths of them? Three fifths of them? Four fifths of them? Five fifths of them?

Quick answers:

Four and one are how many?
Three and two?
Five less four?
Five less two?
Five less five?
Two and three?
One and four?
Five less one?
Two and two and one?
What is one fifth of five?

The sign − means *less*.

Read: $3-1=2$; $4-1=3$; $4-2=2$; $5-1=4$.

Copy and complete these (putting them on the board):

$5-4=$ $4-2=$ $5-1=$ $3-1=$ $3-2=$ $1+4=$
$5-2=$ $4-3=$ $5-3=$ $4-1=$ $2+1=$ $2+3=$

Read them after they are completed.

FOR STUDY.

Make five short lines on your slate. Now erase five of them. How many are left?

Five lines less five lines are how many lines?

If George has five cakes in his dinner pail and eats five, how many will be left in his pail?

Make five circles on your slate. Draw a line through two of them. Draw a line through another two of the circles. How many twos have you drawn a line through? How many circles are left over?

How many twos are there in five? How many are left over?

Two boys and two boys and one boy are how many boys?

Lucy has two apples, Ruth has two apples, and Frances has one apple. How many apples have the three girls all together?

Roy paid two cents for ink, two cents for a top, and one cent for a pen. He gave the clerk a nickel. How many cents will the clerk give back?

Copy and fill blank spaces:

3 cherries and 2 cherries are —— cherries.
5 dogs less 1 dog are —— dogs.
4 books less 2 books are —— books.
1 pencil and —— pencils are 5 pencils.
3 birds + 1 bird = —— birds.
2 rabbits + 3 rabbits = —— rabbits.
2 hands + 2 hands + 1 hand = —— hands.
3 children + —— children = 5 children.

2+2=   1+4=   2+2+1=

LESSON 11.

ORAL EXERCISE.

Tell me quickly in one word the number of splints I hold up.

(The pupils should be drilled on this till they can recognize and answer instantly without having to count the objects.)

Notice what I show you and give the result quickly. (Here the teacher should put one splint with three splints, two splints with three splints, etc.; and take one splint from two splints, two splints from five splints, etc.) **Example**: 1 splint + 2 splints = 3 splints.

Show me one half of two splints. One third of three splints. One half of four splints. One fourth of four splints. One fifth of five splints.

(The teacher should see that each pupil takes the whole number of objects before he shows the part asked for. Simply to show one object taken from a pile of any number of objects does not show one half of two any more than it does one fiftieth of fifty.)

Show me the number of splints these figures (making them on the board) ask for: 3, 5, 1, 4, 2, 5, 4, 5, 1.

2+1
2−1
1+3
3−1
3+2
5−2
2+3
4−2
2+2
5−3
4+1
5−1
2+3
5−4
1+4

Ask frequently: Where are the splints? Where are the figures that stand for them?

Pass to the board.

Make the figures which will tell me the number of splints I show you. (Show the splints and require the figures to be made as quickly as it can be done without confusion.)

Notice what I show you and write the result. (The teacher should again put one splint with two splints; take two splints from five splints, etc.)

Frank bought four apples and gave one half of them to his little brother. How many apples did his brother get? How many apples had Frank left?

John earned five cents weeding the garden. He gave one cent to his little sister, and two cents to his brother. How many cents had he left?

FOR STUDY.

John has three marbles and Frank has two. How many marbles have both boys together? How many more has John than Frank?

Three marbles and two marbles are how many marbles?

Two boys and three boys are how many boys?

How many wings have two hens?

A hen has two white chickens, two black ones, and one gray one. How many chickens has she in all?

Lucy's mother gave her a nickel and sent her to the post office to get a two-cent stamp. How many cents ought Lucy to bring back to her mother?

John has carried in three armfuls of wood. How many more armfuls must he carry in to have five armfuls of wood carried in?

Ned's father has five cows. One fifth of the number are Jersey cows. How many Jersey cows has Ned's father?

Copy and fill blank spaces:

2 buttons + 1 button = —— buttons.
4 spools + —— spool = 5 spools.
2 girls + —— girls = 5 girls.
1 book + 3 books = —— books.
4 birds − 2 birds = —— birds.
5 cows − 3 cows = —— cows.
2 eggs + —— eggs = 5 eggs.

2+2=
2+3=
2−2=
5−1=
3+2=
4−3=
4−1=
5−4=

LESSON 12.

ORAL EXERCISE.

Show me as many splints as there are children in the picture.

How many have you? What are they?

Show me six fingers; six grains of corn, etc.

How many? (showing six of several different kinds of objects successively, and asking the question, "How many?" then, "What kind?" each time).

Lewis may take six steps.

Harry may clap his hands six times.

One boy and five girls are how many? —— what?

Dan may take five nails and one nail, and tell a story about five and one.

Lucy bought one yard of yellow ribbon and five yards of green ribbon. How many yards of ribbon did she buy?

Take six nails. Put one of them on the table. How many nails have you left?

Six nails less one nail are how many nails?

Horace may make six crosses on the board. George may erase five of them. How many crosses are left?

Six crosses less five crosses are how many crosses?

Andrew may take six splints and tell a story about six less five.

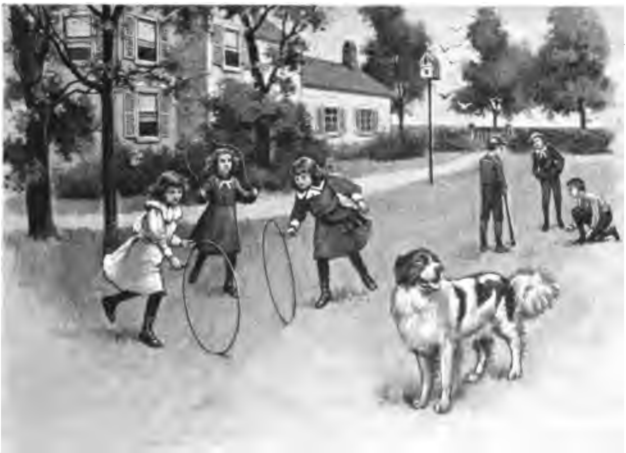
There is one book on Hattie's desk. How many more books must be put on her desk to make six books on her desk?

Laura practices her music six hours a day. After she has practiced five hours, how many more hours must she practice?

One shell and how many shells make six shells? Show me.

Quick answers:

One and five are how many?
Six less five are how many?
Three and how many are five?
Six less one are how many?
Five and how many are six?
Two and two are how many?
Five less two are how many?
One and how many are six?



6 FOR STUDY. **Six**

What do you see in this picture?

How many children are there?

How many trees?

How many windows can you see?

How many feet have all the boys together? Count them.

How many hands have all the girls together?

Make as many circles on your slate as there are trees in the picture. How many circles have you made?

Make as many triangles on your slate as there are children in the picture. How many have you made? What are they?

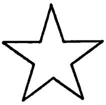
Make five squares on your slate in a row. Make one more square beneath the five squares. How many have you made? What are they?

Five squares and one square are how many squares?

One girl and five girls are how many girls?

Make six dots on your slate. Erase one of them. How many dots are left?

Six dots less one dot are how many dots?



Make six stars like this one on your slate. Draw a line through five of them. How many stars are left without a line through them?

Six stars less five stars are how many stars?

Six stars less one star are how many stars?

John's father bought one calf of one man and five calves of another man. How many calves did John's father buy in all?

Myrtle is learning a poem of six stanzas. She has memorized one stanza. How many more stanzas has she to memorize?

Five pigs and how many pigs make six pigs?

Andrew owed Frank six cents and gave him five cents. How many cents did he still owe Frank?

When the cook has made one pie, how many more must she make to have six pies?

Copy and fill blank spaces:

5 hens and 1 hen are —— hens.
1 pear and —— pears are 6 pears.
6 pens less 5 pens are —— pen.
3 horses and —— horses are 5 horses.
2 bells and —— bells are 4 bells.

The figure that means six is at the beginning of this lesson. Make six of them in a row as neatly as you can.

LESSON 13.

ORAL EXERCISE.

Tell me quickly how many? (holding up six objects. Then review by showing the numbers studied already).

Pass to the board.

Make the figures which tell me how many splints. Where is the number? (Take care that the child does not show the *figure.*) What is the number? What kind is the number? Where is the figure that tells what the number is? (Take seats.)

Read what I write and give the result:

$3+2=$ $6-2=$ $5-1=$ $6-4=$
$4-2=$ $6-5=$ $5-3=$ $6-1=$

Each of you may hold up four fingers on one hand and two fingers on the other. Put them all together. How many fingers are you holding up?

Four fingers and two fingers are how many fingers?

Two fingers and four fingers are how many fingers?

Ethel may tell a story about four and two, with splints.

Show me six splints. Lay two of them on the table. How many have you left?

Six splints less two splints are how many splints?

Luther may take six splints and tell a story about six less two.

All pass to the board.

Make six nice squares in a row. Now erase four of them. How many squares are left?

Ben may tell what he did.

Six boys started down hill on a sled. Two of them fell off on the way down. How many were on the sled when it reached the foot of the hill?

How many wheels have a wagon and a cart together?

Quick answers:

Four and two are how many?
Six less one?
Three and two?
One half of four?
One fifth of five?
Six less four?

Four less three?
Two and four?
Six less five?
One and five?
Six less two?

FOR STUDY.

Make four triangles on your slate. Make two more triangles beneath the four triangles. How many have you made in all? What are they?

Four triangles and two triangles are how many triangles?

Two triangles and four triangles are how many triangles?

Make six straight lines on your slate. Erase two of them. How many lines are left?

Six lines less two lines are how many lines?

Six books less two books are how many books?

Make six circles on your slate. Erase four of them. How many circles are left?

Six circles less four circles are how many circles?

Six pins less four pins are how many pins?

Harry wants a ball which costs six cents. He has only four cents. How many cents must he earn before he can buy the ball?

Six ducks were on a lake. A hunter shot two of them. How many escaped?

How many feet have you and your dog together?

Lucy needs six buttons for her dress. She has only two. How many more must she get?

Ellen is in school six hours a day. Her little sister gets out of school two hours earlier. How many hours a day is her sister in school?

A fly has six legs. On how many legs does it stand while it cleans its wings with two legs?

Two desks and how many desks are six desks?

Copy and fill blank spaces:

4 boys and —— boys are 6 boys.
2 blocks and —— blocks are 6 blocks.
6 chairs less —— chairs are 2 chairs.
4 plums and —— plum are 5 plums.
4 cows and —— cows are 6 cows.
3 slates and —— slates are 5 slates.

$2+3=$
$4-2=$
$6-1=$
$6-2=$
$6-5=$
$5-4=$
$4+\quad=6$
$5-\quad=3$
$6-\quad=2$

LESSON 14.

ORAL EXERCISE.

Show me three fingers on one hand and three fingers on the other. How many in all? Six what?

Show me six splints. Lay three of them on the table. How many are left?

Carrie may place six children in a row. She may stand in the middle of the row. How many children are at her right hand? How many at her left hand? Two times three children are how many children? What is one half of six children?

You may all take six splints. Show me how many threes in six splints. Where is one three? What kind is it? Where is the other three? What kind is it? Show me one half of six splints. One half of six splints is how many? —— what?

Take six nails. Put them in three equal piles. How many nails in each pile? How many twos in six nails? Show me one two. What kind is it? Show me another two. What kind is it? What is one third of six nails? Show me. Show me another one third. Another one third.

Mary bought six yards of ribbon. She gave half of the ribbon to her sister. How many yards did her sister receive? How many yards had Mary left?

Tom earned six cents running an errand. He paid one third of them for a top. How many cents did he pay for the top? How many cents had he left?

Quick answers:

Six less six are how many?
Three and three?
Two and three?
Four and two?
Six less three?
Six less one?
Two and four?
Two and two?
One and five?
One half of six?
One third of six?
How many twos are there in six?
How many threes are there in six?
Two times three are how many?

FOR STUDY.

Make three squares on your slate in a row. Make three more beneath them. How many have you made? What are they?

Three squares and three squares are how many squares?

Three boys and three boys are how many boys?

Make six dots on your slate. Erase three of them. How many dots are left?

Six dots less three dots are how many dots?

Six girls less three girls are how many girls?

Albert can jump three feet. His brother, who is older, can jump three feet farther than he. How many feet can his brother jump?

Frances had six cents. She paid three cents for needles. How many cents had she left?

Make six straight lines on your slate. Erase six of them. How many are left?

Six lines less six lines are how many lines?

Lucy had six raisins. She gave three of them to her sister and three to her playmate. How many raisins had she left?

Make six circles on your slate in a row. Draw a line between the two in the middle. How many circles on each side of the line?

How many threes are there in six circles?

Two threes are how many?

Two times three are how many?

What is one half of six circles?

Make six dots on your slate in a row. Draw a line through two of them. Draw a line through another two. Draw a line through another two. How many twos have you drawn a line through?

How many twos in six dots?

Three times two dots are how many dots?

What is one third of six dots?

Eddie had six cents. He gave one half of them for a pencil. How many cents did he pay for the pencil? How many had he left?

Edna had six cherries. She gave one half of them to her brother. How many cherries did she give her brother? How many had she left?

Copy and fill blank spaces:

6 chairs less —— chairs are no chairs.
3 figs and —— figs are 6 figs.
One half of 6 slates is —— slates.
One third of 6 books is —— books.
6 birds less 3 birds are —— birds.
There are —— threes in 6.
There are —— twos in 6.

LESSON 15.

ORAL EXERCISE.

How many horses are there in the picture?

Show me as many splints as there are horses in the picture.

How many have you shown? Seven what?

Show me seven fingers, etc.

Belle may make seven squares on the board. Fred may take seven steps.

How many do I hold up? —— what? (Pencils.) How many? —— what? (Books), etc.

Show me six splints and one splint. Put them together. How many splints have you?

Six splints and one splint are how many? —— what?

One splint and six splints are how many? —— what?

Robert may tell a story about one and six.

Show me seven splints. Put one of them on the table. How many splints have you left?

Seven splints less one splint are how many splints?

Where is the six? What kind is it? Where is the one? What kind is it?

Elmer may bring me seven crayons. Laura may take six of them. How many have I left?

Seven crayons less six crayons are how many crayons?

Four crayons and how many more crayons make seven crayons?

Bessie has seven problems to solve. How many will be left unsolved after she has solved six of them?

Six problems and one problem are how many problems?

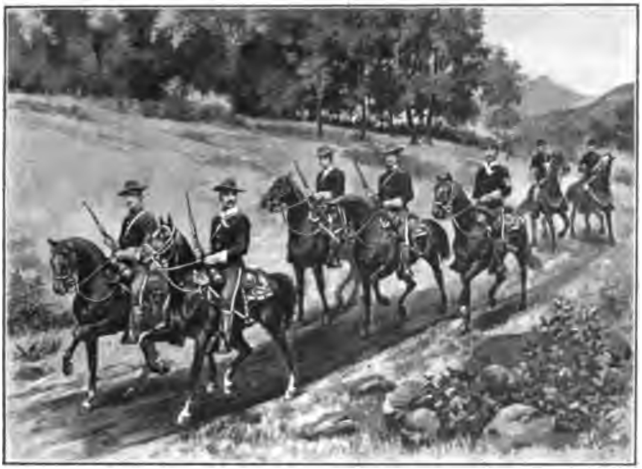
There were seven dolls in a show window. Flora bought one of them. How many were left in the window? —— what?

Six melons and how many melons are seven melons?

Quick answers:

Six and one are how many?
Six less one?
Seven less one?
Seven less six?
Four and two?
Three and two and one?
One is what part of five?

One and six?
One half of two?
One third of three?
One half of four?
One fourth of four?



7 FOR STUDY. **Seven**

What have we in this picture?

How many soldiers are there? Count them.

How many horses?

How many letters in the word *soldier?*

Count the letters in the word picture.

Are there seven letters in your first name? In your last name?

Write your full name neatly seven times on your slate.

Write the names of seven of your schoolmates. Count them.

Make six squares in a row on your slate. Make one more just beneath them. How many have you made in all? What are they?

Six squares and one square are how many squares?

LESSON 12.

ORAL EXERCISE.

Show me as many splints as there are children in the picture.

How many have you? What are they?

Show me six fingers; six grains of corn, etc.

How many? (showing six of several different kinds of objects successively, and asking the question, "How many?" then, "What kind?" each time).

Lewis may take six steps.

Harry may clap his hands six times.

One boy and five girls are how many? —— what?

Dan may take five nails and one nail, and tell a story about five and one.

Lucy bought one yard of yellow ribbon and five yards of green ribbon. How many yards of ribbon did she buy?

Take six nails. Put one of them on the table. How many nails have you left?

Six nails less one nail are how many nails?

Horace may make six crosses on the board. George may erase five of them. How many crosses are left?

Six crosses less five crosses are how many crosses?

Andrew may take six splints and tell a story about six less five.

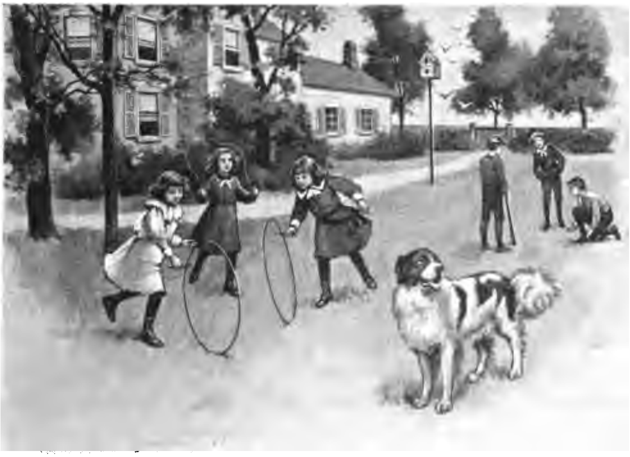
There is one book on Hattie's desk. How many more books must be put on her desk to make six books on her desk?

Laura practices her music six hours a day. After she has practiced five hours, how many more hours must she practice?

One shell and how many shells make six shells? Show me.

Quick answers:

One and five are how many?
Six less five are how many?
Three and how many are five?
Six less one are how many?
Five and how many are six?
Two and two are how many?
Five less two are how many?
One and how many are six?



6 FOR STUDY. **Six**

What do you see in this picture?

How many children are there?

How many trees?

How many windows can you see?

How many feet have all the boys together? Count them.

How many hands have all the girls together?

Make as many circles on your slate as there are trees in the picture. How many circles have you made?

Make as many triangles on your slate as there are children in the picture. How many have you made? What are they?

Make five squares on your slate in a row. Make one more square beneath the five squares. How many have you made? What are they?

LESSON 12.

ORAL EXERCISE.

Show me as many splints as there are children in the picture.

How many have you? What are they?

Show me six fingers; six grains of corn, etc.

How many? (showing six of several different kinds of objects successively, and asking the question, "How many?" then, "What kind?" each time).

Lewis may take six steps.

Harry may clap his hands six times.

One boy and five girls are how many? —— what?

Dan may take five nails and one nail, and tell a story about five and one.

Lucy bought one yard of yellow ribbon and five yards of green ribbon. How many yards of ribbon did she buy?

Take six nails. Put one of them on the table. How many nails have you left?

Six nails less one nail are how many nails?

Horace may make six crosses on the board. George may erase five of them. How many crosses are left?

Six crosses less five crosses are how many crosses?

Andrew may take six splints and tell a story about six less five.

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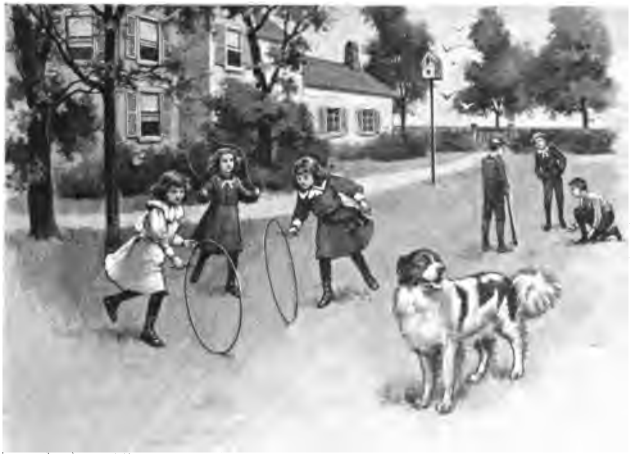

6 **FOR STUDY.** **Six**

What do you see in this picture?

How many children are there?

How many trees?

How many windows can you see?

How many feet have all the boys together? Count them.

How many hands have all the girls together?

Make as many circles on your slate as there are trees in the picture. How many circles have you made?

Make as many triangles on your slate as there are children in the picture. How many have you made? What are they?

Make five squares on your slate in a row. Make one more square beneath the five squares. How many have you made? What are they?

LESSON 12.

ORAL EXERCISE.

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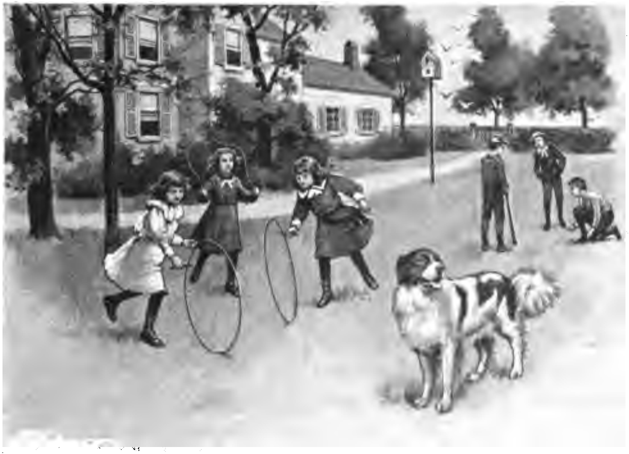

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What do you see in this picture?

How many children are there?

How many trees?

How many windows can you see?

How many feet have all the boys together? Count them.

How many hands have all the girls together?

Make as many circles on your slate as there are trees in the picture. How many circles have you made?

Make as many triangles on your slate as there are children in the picture. How many have you made? What are they?

Make five squares on your slate in a row. Make one more square beneath the five squares. How many have you made? What are they?

LESSON 12.

ORAL EXERCISE.

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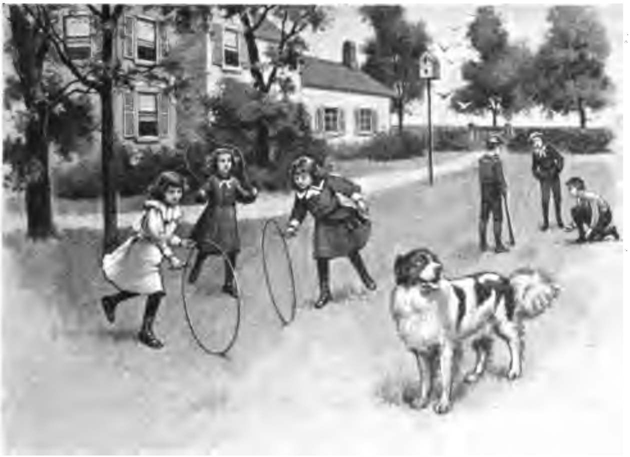

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How many children are there?

How many trees?

How many windows can you see?

How many feet have all the boys together? Count them.

How many hands have all the girls together?

Make as many circles on your slate as there are trees in the picture. How many circles have you made?

Make as many triangles on your slate as there are children in the picture. How many have you made? What are they?

Make five squares on your slate in a row. Make one more square beneath the five squares. How many have you made? What are they?

LESSON 12.

ORAL EXERCISE.

Show me as many splints as there are children in the picture.

How many have you? What are they?

Show me six fingers; six grains of corn, etc.

How many? (showing six of several different kinds of objects successively, and asking the question, "How many?" then, "What kind?" each time).

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Six nails less one nail are how many nails?

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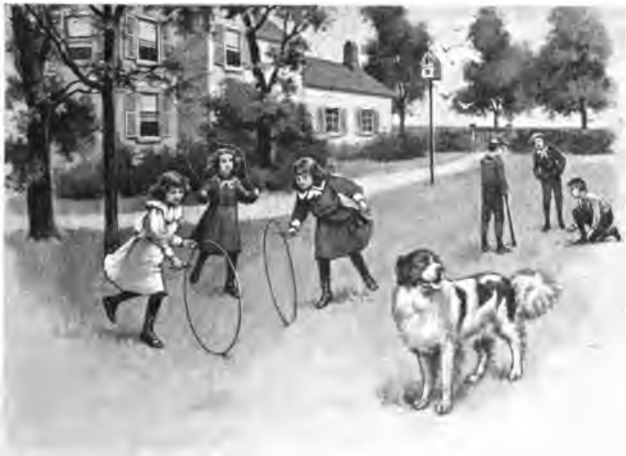

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How many children are there?

How many trees?

How many windows can you see?

How many feet have all the boys together? Count them.

How many hands have all the girls together?

Make as many circles on your slate as there are trees in the picture. How many circles have you made?

Make as many triangles on your slate as there are children in the picture. How many have you made? What are they?

Make five squares on your slate in a row. Make one more square beneath the five squares. How many have you made? What are they?

One half of 6 children is —— children.
In 7 there are —— twos and —— left over.
6 cents will buy —— two-cent postage stamps.

$4+3=$
$7-1=$
$6-3=$
$5+2=$
$6-2=$
$7-5=$
$7-7=$
$7-3=$
$4+2=$
$2+2+1=$
$3+3+1=$
$4+3-2=$

Make the figure 8, and write the word *eight*, neatly, eight times.

LESSON 20.

ORAL EXERCISE.

Show me the number of splints these figures ask for: 8, 6, 8, 7, 5, 8, 1, 8, 4, 3, 8.

Make the figure which tells me the number I hold up. (Let the teacher follow each number of objects with the questions, "What is the number?" "Where is the number?" "What kind is it?" "Where is the figure?" "What does it stand for?")

Put two splints with six splints. How many?

Six splints and two splints are how many splints?

Take eight splints. Lay two of them on the table. How many are left?

Take six splints from eight splints. How many are left?

Ralph had eight rows of potatoes to hoe. John hoed two rows for him. How many rows did Ralph hoe?

Quick answers:

Six and two are how many?
Eight less one?
One and seven?
Eight less two?
Six less two?
Five less three?
One half of four?
One fourth of four?
One fifth of five?
Three and four?
Seven less four?
Three and three?
Six less four?
Four and five?
One third of six?
Eight less six?
Two and six?

FOR STUDY.

Make six triangles on your slate in a row. Make two more triangles beneath them. How many triangles have you made in all?

Six triangles and two triangles are how many triangles?

Two triangles and six triangles are how many triangles?

On our street there are six two-story houses and two cottages. How many houses are there on our street?

Make eight squares on your slate. Erase two of them. How many squares are left?

Eight squares less two squares are how many squares?

There were eight cherries on a twig. How many were left after a robin ate two of them?

Make eight dots on your slate. Erase six of them. How many dots are left?

Eight dots less six dots are how many dots?

Harry carried eight quarts of cherries to market. He sold all but two quarts. How many quarts did he sell?

Jane bought some soap and some matches. She paid six cents for the soap and enough for the matches to make eight cents. How many cents did she pay for the matches?

Two new pupils joined Ethel's Sunday-school class last Sunday. There are now eight pupils in the class. How many were there before?

How many cents added to two cents will make eight cents?

John is eight years old and Ernest is two years younger. How old is Ernest?

Copy and fill blank spaces:

2 tops + 5 tops = —— tops.
8 pictures − 6 pictures = —— pictures.
7 baskets − —— baskets = 3 baskets.
6 goats + —— goats = 8 goats.
5 bananas + —— bananas = 7 bananas.
2 lemons + —— lemons = 8 lemons.
1 squirrel + —— squirrels = 8 squirrels.

3+4=
4+2=
6+2=
7+1=
8−2=
2+3=
5−2=
4−3=
8−6=
8−1=
6−4=
8−7=
2+2=
3+3=
4+4=
2+2+2=

LESSON 21.

ORAL EXERCISE.

Six apples and two apples are how many apples?

Eight doves less one dove are how many doves?

Eight pins less six pins are how many pins?

Show me five shells. Show me three more. Put them together. How many have you in all? —— what?

See if three beans and five beans are eight beans.

A farmer sleeps three hours before midnight and five hours after midnight. How many hours does he sleep?

Show me eight shells. Lay three of them on the table. How many are left?

Show me eight splints. Put five of them behind you. How many are in sight?

Eight horses less three horses are how many horses?

How many are eight balls less five balls?

Edgar earned five cents by running an errand and three cents by holding a horse. How many cents did he earn? Tell the story with splints.

A pail holds eight quarts. When three quarts of milk have been put into it, how many more must be put into it to fill it?

Quick answers:

Five and three are how many?
Eight less two?
One and seven?
Eight less three?
Two and six?
Eight less seven?
Three and five?
Eight less six?
Six and two?
Eight less five?
Three and three?
Four and three?
Five and two?

How many threes are there in seven? How many left over? Show me with splints.

FOR STUDY.

Make five circles on your slate in a row. Make three more circles beneath them. How many circles have you made?

Five circles and three circles are how many circles?

Three circles and five circles are how many circles?

Nellie is five years old. How old will she be in three more years?

Make eight dots on your slate. Erase three of them. How many dots are left?

Eight dots less three dots are how many dots?

Eight fingers less three fingers are how many fingers?

I owe Mary eight cents. After paying her a nickel, how many cents shall I owe her?

Make eight straight lines in a row. Erase five of them. How many are left?

Eight lines less five lines are how many lines?

Eight pins less five pins are how many pins?

There were eight cherry trees in my orchard. The wind blew five of them down. How many cherry trees were left?

In three years Frank will be eight years old. How old is he now?

Five boys started down hill on a sled. When the sled reached the foot of the hill there were eight boys on it. How many got on after the sled started down hill?

A post was set three feet in the ground. How many feet must there be in the post above ground if its whole length is eight feet?

Frances has eight pages of music to learn. How many pages does she still have to learn after she has learned five pages?

Copy and fill blank spaces:

8 pigeons less —— pigeons are 5 pigeons.
5 melons + 2 melons = —— melons.
6 pies + —— pies = 8 pies.
3 foxes + —— foxes = 8 foxes.
8 baskets − 2 baskets = —— baskets.
1 window + —— windows = 8 windows.
2 balls + —— balls = 8 balls.

3+4=
8−3=

6+2=
7−3=

8−6=
3+3=

3+5=
6−4=

In 6 are —— twos.
One half of 6 is ——.

In 6 are —— threes.
One third of 6 is ——.

Tell a story about:

5 cats and 3 cats; 8 cents less 6 cents; 7 eggs less 4 eggs; 8 boys less 5 boys.

LESSON 22.

ORAL EXERCISE.

Show me four fingers on the right hand. Show me four on the left hand. How many on both? —— what?

How many are four oranges and four oranges?

Two times four oranges are how many?

Eight children less four children are how many children?

You may all take eight splints. Show me one half of eight splints. Show me another one half of eight splints. How many splints is one half of eight splints?

Four boys are what part of eight boys?

What is one half of eight books? One half of eight horses?

You may all lay your splints in four equal piles. How many splints are there in each pile? How many twos are there in eight? Show me one two. What kind is it? Show me another two, etc.

Show me one fourth of eight splints. What is one fourth of eight splints? Show me another fourth, etc.

What is one eighth of eight?

Quick answers:

Eight less eight are how many?
Eight less four?
Three and five?
Three and four?
Eight less six?
One half of eight?
Six and two?
One fourth of four?
Four times two?
Two times four?
Eight less five?
One and seven?
One eighth of eight?
How many twos are there in eight?
How many fours are there in eight?
Count by twos to eight.

FOR STUDY.

Make four squares in a row on your slate. Make four more squares beneath them. How many squares have you made in all?

Four squares and four squares are how many squares?

How many times four squares are there in eight squares?

Two times four squares are how many squares?

What is one half of eight squares?

Erase four squares. How many squares are left?

Eight squares less four squares are how many squares?

Make eight crosses on your slate, in a row. Draw a line through two of them. Draw a line through another two. Do the same with two more of them. Draw a line through another two of them. Through how many twos have you drawn lines?

How many twos in eight crosses?

Four times two crosses are how many crosses?

What is one fourth of eight crosses?

You have eight crosses. Erase eight crosses. How many crosses are left?

Eight crosses less eight crosses are how many crosses?

David had eight cents. He paid four cents for a top and four cents for raisins. How many cents had he left?

A wagon and a carriage have how many wheels together?

Ned's father gave him eight pigs. He sold one half of them. How many pigs did he sell?

How many two-cent postage stamps can I buy for eight cents?

A gardener sets out trees four in a row. How many rows can he make with eight trees?

Rice is worth eight cents a pound. How many cents will one half a pound cost?

Eight children are to sing in a chorus. Four of them are girls and the remainder are boys. How many boys will sing?

Eight wheels will supply how many bicycles?

Copy and fill blank spaces:

8 cents will buy —— two-cent postage stamps.
One half of 8 marbles is —— marbles.
4 desks and —— desks are 8 desks.
8 wheels will supply —— wagons.
One fourth of 8 pencils is —— pencils.
8 nuts less —— nuts are no nuts.

LESSON 23.

ORAL EXERCISE.

Show me one half of eight; one fourth of eight. How many twos are there in eight? How many fours are there in eight?

How many lilies in the picture on the following page?

Show me nine shells; nine children.

How many? (Nails.) What kind?

Take eight splints and one splint. Put them together. How many? —— what?

Roy may tell a story about one and eight.

Hold up nine splints. Take one away.

Mamie may tell a story about nine less one.

Hold up nine splints. Take eight away.

Carrie may tell a story about nine less eight.

Nine crows were on the fence. Eight of them flew away. How many were left?

Amos trapped nine rabbits in one day. He sold all but one. How many did he sell?

Quick answers:

What is one half of eight?
What is one fourth of eight?
Seven less four?
Five and three?
Two and four?
Three less three?
Eight less four?
Six and two?
Two and five?
One and seven?
Six less three?
Three and five?
Nine less eight?
Seven less seven?
One and four?
Four and three?

FOR STUDY.

What have we in this picture? How many lilies are there? Count them.

Make three triangles on your slate. How many sides have they all together?

Make eight circles in a row. Make one more circle beneath them.

How many circles are eight circles and one circle?

How many circles are one circle and eight circles?

There were eight crows on a fence and one more crow joined them. How many were on the fence then? —— what?

Ben had one marble. His uncle gave him a bag containing eight marbles. How many marbles had he then?

Make nine squares in a row on your slate. Erase one of them. How many squares are left?

Nine squares less one square are how many squares?

Make nine straight lines in a row. Erase eight of them. How many are left?

Nine lines less eight lines are how many lines?

There were nine cherries on one twig. On breaking the twig, one cherry dropped off. How many cherries were left on the twig?

Frank had nine cents. How many cents had he left after paying eight cents for a kite?

If John has picked eight quarts of blackberries, how many more quarts must he pick to have nine quarts?

Herbert has nine pigs. How many pigs must he sell to have only one left?

Mary paid one cent for a pencil and enough more for a tablet to make nine cents. How many cents did the tablet cost?

The figure that means nine is at the left of the lilies. Make it neatly nine times. Write the word *nine* nine times.

Copy and fill blank spaces:

8 birds and 1 bird are —— birds.
9 squirrels less 1 squirrel are —— squirrels.
5 stars and —— stars are 8 stars.
3 balls and —— balls are 7 balls.
1 lemon and —— lemons are 9 lemons.
One half of 8 hats is —— hats.
2 geese and —— geese are 8 geese.
9 pencils less 8 pencils are —— pencil.

3+4=
9−8=
5+3=

8−6=
5+2=
8−4=

1+8=
8+3=
2+6=

7−4=
3+3=
9−1=

LESSON 24.

ORAL EXERCISE.

Take seven splints. Put two more splints with them.

Julia may tell what she did.

Take two splints and put seven more with them.

Hold up nine splints. Take two of them away.

Show nine beans. Take away seven beans. How many are left? —— what?

Clay may tell a story about nine less seven.

Lizzie had nine peaches and gave two to a playmate. How many had she left?

Minnie visited her cousins a week and two days. How many days did she visit them?

Robert is nine years old. His brother is two years younger. How old is his brother?

Ellen is seven years old. In how many more years will she be nine?

Quick answers:

Nine less two are how many?
Five less four?
Two and six?
Two and seven?
Eight and one?
Nine less seven?
Seven less seven? etc.

FOR STUDY.

Make seven circles on your slate in a row. Make two more circles beneath them.

How many circles are seven circles and two circles?

How many hats are two hats and seven hats?

Mr. Jones has two cows. One of them gives two quarts of milk a day, and the other gives seven quarts. How many quarts do both cows give in a day?

Make nine stars in a row on your slate. Erase two of them. How many stars are left?

Nine stars less two stars are how many stars?

Make nine dots in a row. Erase seven of them. How many dots are left?

Nine dots less seven dots are how many dots?

Nine dots less two dots are how many dots?

Alice had nine words to learn. After learning seven, how many more did she have to learn?

Frank had nine pears. He gave four to Tom and three to Albert. How many had he left?

I need nine postal cards, and I have but two. How many must I buy?

Willie has learned two stanzas of a poem, and he has seven more stanzas to learn. How many stanzas are in the poem?

Ralph had nine arrows and lost all but two of them. How many did he lose?

Helen paid five cents for ink, two cents for paper, and two cents for pens. How many cents did she pay for all?

Copy and fill blank spaces:

9 oxen less 2 oxen are —— oxen.
1 rose and —— roses are 9 roses.
2 lambs and —— lambs are 9 lambs.
3 flies and —— flies are 8 flies.
9 arrows less 2 arrows are —— arrows.
2 carts and —— carts are 8 carts.
9 cherries less —— cherries are 2 cherries.
In 8 mittens there are —— pairs of mittens.
7 books and —— books are 9 books.

$7+2=$ $9-1=$ $2+5=$ $8-6=$ $8-8=$
$5+3=$ $9-7=$ $1+8=$ $2+7=$ $9-3=$

LESSON 25.

ORAL EXERCISE.

Take nine splints.

Who can put them in three equal piles? Harry, how many splints have you in each pile?

You may all put three splints in a pile.

How many threes in nine? Show me one three. What kind? Show me another three. What kind? Three times three splints are how many? —— what? Count by threes to nine.

Show me one third of nine splints. How many splints in one third of nine splints? Show me another third of nine splints. Another third.

How many splints are there in two of these piles?

Six splints and three splints are how many? —— what?

Three splints and six splints are how many splints?

Nine splints less three splints are how many splints? Show me.

Nine splints less six splints are how many splints? Show me.

Six splints and how many splints make nine? —— what?

Quick answers:

How many threes are there in nine?
Six and three?
Nine less two?
Two and seven?
Nine less eight?
Three and six?
One third of nine?
One and eight?
Three times three?
One fourth of eight?
Three and three and three?
Nine less one? etc.

How many three-cent postage stamps can I buy with nine cents?

FOR STUDY.

Make three squares in a row. Make three more squares in a row just beneath the first three. How many squares have you made?

Three squares and three squares are how many squares?

Three fingers and three fingers are how many fingers?

Make three more squares in a row just beneath the squares you have made.

How many squares have you made in all?

How many rows of squares have you made? How many squares in each row?

Three squares and three squares and three squares are how many squares?

Three times three squares are how many squares?

How many threes are there in nine squares? How many threes in nine fingers?

What is one third of nine squares?

What is one third of nine fingers? Can you show it?

Mary bought nine bananas. She gave one third of them to an old blind lady. How many bananas did she give away?

Make six dots in a row. Make three more dots beneath them. How many dots have you made?

Six dots and three dots are how many dots?

Three dots and six dots are how many dots?

Arthur picked three quarts of berries in the morning before school and six quarts after school. How many quarts of berries did he pick?

Fannie found six eggs in one nest and three eggs in another. How many eggs did she find?

You have nine dots on your slate. Erase three of them. How many dots are left?

Nine dots less three dots are how many dots?

Nine children started down hill on a sled. When the sled reached the foot of the hill there were only six children on it. How many fell off?

Make nine straight lines. Erase six of them. How many are left?

Nine lines less six lines are how many lines?

Nine squares less three squares are how many squares?

There were nine peach trees in my orchard. The winter killed six. How many were left alive?

Fred and Frank went hunting. Fred killed six ducks and Frank killed enough to make nine ducks. How many ducks did Frank kill?

Fannie went shopping with nine cents. She paid three cents for thread and three cents for a thimble. How many cents had she left?

A gardener has nine trees. He plants three trees in a row. How many rows are there?

Agnes plucked nine wild flowers on her way to school. She gave one third of them to her teacher. How many flowers did she give to her teacher? How many had she left?

Copy and fill blank spaces:

6 lions and 3 lions are —— lions.
9 figs less 2 figs are —— figs.
1 ball and —— balls are 9 balls.
One third of 9 oranges is —— oranges.
3 lilies and —— lilies are 9 lilies.
9 bells less 7 bells are —— bells.
Three times 3 pencils are —— pencils.
7 plums and —— plums are 9 plums.
9 doves less 3 doves are —— doves.

$6+3=$	$9-6=$	$9-1=$	$1+8=$
$9-2=$	$2+7=$	$3+6=$	$9-3=$
$3+3=$	$4+3=$	$6-4=$	$4+4=$

LESSON 26.

ORAL EXERCISE.

(A short, rapid review of the preceding lesson.)

Show me five splints. Now four. Put them together. Five and four are how many? Four and five?

Hold up nine splints. Take away four. How many are left?

Put the four with the five again. Now how many have you? —— what?

Take away five. How many have you left?

Quick answers:

Five and four are how many?
Three and six?
Two and seven?
Nine less three?
One half of eight?
Nine less five?
Nine less seven? etc.

Harold gathered four pecks of nuts and Thomas gathered five. How many did both boys gather? —— what?

Five children started down hill on a sled. When the sled reached the foot of the hill there were nine children on it. How many got on after the sled started?

Four cents and how many cents make nine cents?

FOR STUDY.

Make five triangles in a row. Make four more triangles just beneath them. How many triangles have you made?

Five triangles and four triangles are how many triangles?

Four triangles and five triangles are how many triangles?

Five cakes and four cakes are how many cakes?

Four cakes and five cakes are how many cakes?

Emma has five white roses and four yellow ones. How many roses has she?

Make nine crosses. Erase four of them. How many are left?

Nine crosses less four crosses are how many crosses?

Nine fingers less four fingers are how many fingers? Can you show it?

Richard found nine eggs in one nest. He broke four of them while carrying them to his mother. How many eggs were left?

Make nine dots. Erase five of them. How many are left?

Nine dots less five dots are how many dots?

Nine triangles less five triangles are how many triangles? Can you show it?

Maud bought nine needles. She gave five of them to her sister. How many did she keep?

Ralph has nine marbles and William has four. How many more marbles has Ralph than William?

Katie will need nine spools of thread for the quilt she is making. She has five spools. How many spools must she buy?

George earned a nickel on Monday and four cents on Tuesday. How many cents did he earn both days?

Lucy is four years old and Anna is nine years old. How many years older must Lucy be to be as old as Anna is now?

Karl's mother gave him permission to coast down hill nine times in one day. He coasted five times in the forenoon. How many times may he coast in the afternoon?

Thomas had four cents in the morning, and he earned five cents by holding a horse. How many cents had he left after paying three for nuts?

Copy and fill blank spaces:

9 soldiers less 4 soldiers are —— soldiers.
7 horses and —— horses are 9 horses.
6 goats and —— goats are 9 goats.
4 turkeys and —— turkeys are 9 turkeys.
9 tops less 7 tops are —— tops.
3 sheep are one third of —— sheep.
3 inkstands and —— inkstands are 9 inkstands.
2 melons and —— melons are 9 melons.
One third of 9 hens is —— hens.

4+5=
9−3=
2+7=
9−8=
4+4=
9−5=
6+3=
4+3=
7−5=
9−6=
3+5=
9−2=

LESSON 27.

ORAL EXERCISE.

Show me the number of splints these figures ask for: 8, 1, 9, 3, etc. Where is the number? What is the number? Where is the figure which tells the number?

Make the figures which tell the number I show you (showing several, and asking: Where is the number? Where is the figure that stands for it?).

Quick answers:

Five and three are how many?
Three and six?
Six and two?
Two and seven?
Seven less five?
Nine less four?
Nine less six?
Nine less two?
Nine less eight?

Three times three?
Eight less six?
Three and three?
Four and four?
Two and three?
Seven less four?
Three and three and three?
One half of four?
One third of nine?

FOR STUDY.

John blacked eight boots. How many pairs did he black?

Six dogs were chasing two foxes. How many animals were in the chase?

Four years is one half of Mary's age. How old is she?

Nine boys were playing "fox and geese." Two boys were "foxes." How many "geese" were there?

James has five apples and Oscar has three. How many apples have both together?

Ralph paid five cents for two baskets of shavings. He sold them at four cents a basket. How many cents did he gain by the trade?

How many cents added to a nickel will make nine cents?

Albert and George found eight nuts. How many will each have if they are shared equally?

Lewis and Elmer went fishing. Lewis caught five fish and Elmer two fewer. How many fish did both boys catch?

A blacksmith has eight shoes. How many horses can be shod with them?

Fred spent one half of his money. He had three cents left. How many cents had he at first?

Frank had four cents and earned a nickel by doing an errand. How many cents had he left, after paying three cents for a pencil?

A farmer brought nine bushels of potatoes to market. Ella's mother bought one third of them. How many bushels did she buy?

Walter has eight cents with which to buy apples. They are two cents apiece. How many apples can he buy?

Alice has one half of eight cents and Frances has one half of six cents. Which has the more money? How many cents more?

LESSON 28.

ORAL EXERCISE.

How many fingers, counting the thumbs, have you on both hands?

Show me ten splints; ten beans; ten spools; etc.

Take ten steps; raise and lower the hand ten times; etc. We use two figures to write ten in this way—10.

(Teach 9+1; 1+9; 10−1; 10−9. Do not forget the making and illustrating of "stories" by the pupils.)

If six sheets of paper cost nine cents and a pen one cent, what do the paper and pen cost together?

James had ten apples. He gave all but one to his playmate. How many did he give away?

Mamie lacked one orange of having enough oranges to give one to each of her ten classmates who are spending the afternoon with her. How many oranges had she?

Edith had nine cents in her purse. How many more cents must she put into her purse to have ten cents?

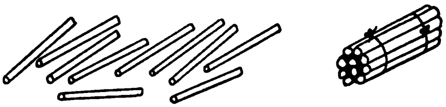
Each show me ten splints. Put a rubber band around them. How many bunches have you? How many splints in a bunch?

Quick answers:

1+9=?
10−9=?
7+2=?
3+4=?
10−1=?
8−3=?

7−4=?
1+9=?
6−3=?
4+4=?
5+4=?
9−6=?

4+2=?
8−6=?
9−2=?
10−7=?
9−7=?
6+3=?



10 FOR STUDY. Ten

Count the loose sticks in the picture. How many are there?

Count the ends of the sticks in the bundle.

How many sticks are there in the bundle?

Count all your fingers and thumbs. How many are there?

How many toes have you on both feet?

Make nine dots in a row. Make one more beneath them. How many dots have you made?

Nine dots and one dot are how many dots?

One dot and nine dots are how many dots?

Make ten circles. Erase one of them. How many are left?

Ten circles less one circle are how many circles?

Make ten straight lines. Erase nine of them. How many are left?

Ten fingers less nine fingers are how many fingers? Can you show it?

Hattie bought one spool of red thread and nine spools of white thread. How many spools of thread did she buy?

A hen has a brood of ten chickens. There is one black one. The rest are white. How many chickens are white?

Harold had ten cents. He paid five cents for nuts and four cents for raisins. How many cents had he left?

Fred earns four cents on Monday, five cents on Tuesday, and enough on Wednesday to make ten cents in all. How many cents does he earn on Wednesday?

Alice picked six quarts of berries, Jennie three quarts, and Flora one quart. How many quarts did all pick?

Ten turtles were basking in the sun. When they saw me, all but one slid into the water. How many slid into the water?

One cent and how many cents make ten cents?

Copy and fill blank spaces:

10 apples less 9 apples are —— apple.
1 thimble and —— thimbles are 10 thimbles.
3 hoes and —— hoes are 9 hoes.
One third of 9 cups is —— cups.
In 6 shoes are —— pairs of shoes.

$4+3+2=$
$3+3+3=$
$7+2+1=$
$10-9+1=$
$3+5-4=$
$6+2-5=$
$7-2-4=$
$2+2+2+2=$
$5+4-6=$

Make the figures for ten *ten* times on your slate.

LESSON 29.

ORAL EXERCISE.

NOTE.—Give a short, rapid review of last lesson.

(Teach $8+2=10$; $2+8=10$; $10-2=8$; $10-8=2$.)

There are eight cherry trees and two plum trees in my garden. How many trees are there in my garden?

Andrew paid eight cents for a bottle of ink. He gave the clerk a dime. How many cents should the clerk return?

Hattie's shoes cost two dollars and her cloak eight dollars. How many dollars did both cost?

Ira spelled eight words and missed enough to make ten words. How many words did he miss?

There are two white roses on one bush and enough red roses on another to make ten roses. How many red roses are there?

Quick answers:

2+8=?
10-9=?
4+5=?
7-3=?
10-8=?
1+9=?
6+3=?
8-5=?
10-2=?
4+3=?
10-1=?
8+2=?

FOR STUDY.

Make eight dots. Make two dots just beneath them. How many dots have you made?

Eight dots and two dots are how many dots?

Two dots and eight dots are how many dots?

Two fingers and eight fingers are how many fingers? Can you show it?

Eight fingers and two fingers are how many fingers?

Make ten circles. Erase two of them. How many circles are left?

Ten circles less two circles are how many circles?

Ten fingers less two fingers are how many fingers? Can you show it?

Make ten dots. Erase eight of them. How many are left?

Ten dots less eight dots are how many dots?

Ten fingers less eight fingers are how many fingers? Can you show it?

Flora's mother baked ten biscuits for breakfast. Eight were eaten. How many were left?

Allen paid five cents for ink, three cents for pens, and two cents for a pencil. How many cents did he pay for all?

Nancy bought eight cents' worth of thread. She gave the clerk a dime. How many cents ought the clerk to give back?

There were ten plum trees in my orchard. The wind blew down eight of them. How many were left standing?

Mary had eight cents and her mother gave her enough to make ten cents. How many cents did her mother give her?

Nathan paid two cents for a pencil and enough for a top to make both cost ten cents. How many cents did the top cost?

Nancy had eight birds in her cage and she put in enough more to make ten birds. How many birds did she put in?

How many wheels have a wagon, a carriage, and a cart?

Copy and fill blank spaces:

10 lambs less 2 lambs are —— lambs.
2 chairs and —— chairs are 10 chairs.
1 quince and —— quinces are 10 quinces.
6 rabbits and —— rabbits are 9 rabbits.
10 boys less 8 boys are —— boys.
8 feet and —— feet are 10 feet.
3 splints and 7 splints are —— splints.
10 pecks less 9 pecks are —— peck.

8+2=	6+3=	5+3=	4+3=	1+9=
10−9=	9−4=	8−6=	7−5=	10−8=

LESSON 30.

ORAL EXERCISE.

(Teach 7+3=10; 3+7=10; 10−3=7; 10−7=3.)

Henry has seven marbles. How many more must he get to have ten marbles?

In a pasture are ten sheep. Three of them are black and the remainder are white. How many white ones are there?

From a class of ten children, seven were taken out and put into a higher class. How many were left?

Florence is trying to get ten dollars to buy a cloak. She has three dollars. How many more must she get?

Quick answers:

7+3=?
10−9=?
2+8=?
5+3=?
10−2=?
1+9=?
3+7=?
4+5=?

FOR STUDY.

Make seven squares in a row. Make three more beneath them. How many squares have you made?

Seven squares and three squares are how many squares?

Three squares and seven squares are how many? —— what?

Seven fingers and three fingers are how many fingers? Can you show it?

Three fingers and seven fingers are how many fingers? Can you show it?

Make ten straight lines. Erase three of them. How many lines are left?

Ten lines less three lines are how many lines?

Make ten stars. Erase seven of them. How many are left?

Ten stars less seven stars are how many stars?

Ten boys were skating. Three of them went home to dinner. How many were left?

There were seven sheep in a pasture. A farmer put three more into the pasture. How many sheep were in the pasture then?

George wants a knife that costs ten cents. He has only seven cents. How many more cents must he earn before he can buy the knife?

Edgar paid three cents for a top. He gave the clerk a dime. How many cents must the clerk return?

Ellen had seven cents and her mother gave her one half of six cents. How many cents had she then?

Alice paid five cents for a thimble, two cents for thread, and three cents for needles. How many cents did she spend?

A train was due at three o'clock. It did not reach the station till ten o'clock. How many hours late was it?

Copy and fill blank spaces:

7 books and —— books are 10 books.
10 oranges less 8 oranges are —— oranges.
3 melons and —— melons are 10 melons.
10 dollars less 9 dollars are —— dollar.
2 carriages and —— carriages are 10 carriages.
10 cows less 3 cows are —— cows.
1 man and —— men are 10 men.

$10-7=$
$9-7=$
$8+2=$
$6+3=$

$2+8-3=$
$9+1-2=$
$4+4-5=$
$10-7+3=$

$10-2+1=$
$1+9-8=$
$6+2+2=$
$3+7-9=$

LESSON 31.

ORAL EXERCISE.

(Teach $6+4=10$; $4+6=10$; $10-4=6$; $10-6=4$.)

A drover bought four pigs of one man and six of another. How many pigs did he buy of both men?

Nancy has a writing book of ten pages. She has written four pages. How many pages has she to write?

Fred had six cents and his father gave him one half of eight cents. How many cents had he then?

Rufus is six years old. How many years must be added to his age to make him ten years old?

There are ten boys and six girls in a class. How many more boys than girls in the class?

Quick answers:

$6+4=?$
$10-3=?$
$2+8=?$
$5+4=?$
$7+3=?$
$10-4=?$
$7-3=?$
$9-5=?$
$10-2=?$
$10-6=?$
$3+7=?$
$4+6=?$

FOR STUDY.

Make six stars in a row. Make four more stars beneath them. How many have you made?

Six stars and four stars are how many stars?

Four stars and six stars are how many stars?

Six fingers and four fingers are how many fingers?

Four fingers and six fingers are how many fingers?

Make ten crosses. Erase four of them. How many crosses are left?

Ten crosses less four crosses are how many crosses?

Make ten dots. Erase six of them. How many dots are left?

Ten dots less six dots are how many dots?

Eugene has six red roses and four yellow ones. How many roses has he?

Laura paid ten cents for thread and six cents for needles. How many more cents did she pay for thread than for needles?

John had ten cents. He gave two cents for a pencil and two cents for an orange. How many cents had he left?

Fannie gave four chestnuts to Carrie and six to Flora. How many did she give both of them?

Lena practiced her music six hours Monday, and enough Tuesday to make ten hours. How many hours did she practice Tuesday?

A grocer had ten barrels of potatoes. He sold four barrels to one man and one half as many barrels to another man. How many had he left?

I owed a man ten dollars. I paid him at one time one third of nine dollars, and at another time one half of six dollars. How many dollars do I still owe him?

Copy and fill blank spaces:

4 sheep and 6 sheep are —— sheep.
7 lambs and —— lambs are 10 lambs.
10 doves less 8 doves are —— doves.
10 pens less 1 pen are —— pens.
4 quail and —— quail are 10 quail.
3 quinces and —— quinces are 10 quinces.
10 snowballs less 6 snowballs are —— snowballs.

5+3−2=
6+4−3=
3+4+3=

10−4+2=
5+4−3=
10−3−4=

3+2+3=
7+3−6=
10−7+6=

LESSON 32.

ORAL EXERCISE.

(Teach 5+5=10; 10−5=5; 2 times 5=10; 5 times 2=10.)

George coasted down hill five times in the forenoon and five times in the afternoon. How many times did he coast down hill that day?

Hattie is ten years old. Lucy's age is one half the age of Hattie. How old is Lucy?

Ten horses make how many pairs of horses?

Grant bought ten oranges, and gave one half of them to his playmates. How many oranges had he left?

Quick answers:

5+5=?
10−6=?
10−7=?
2 times 5=?

4+6=?
2+8=?
3+7=?

10−10=?
6+ 4=?
10− 5=?

FOR STUDY.

Make five squares in a row. Make five more in a row just beneath the first five. How many squares have you made?

Five squares and five squares are how many squares?

How many squares in each row of squares?

How many rows of five squares each?

How many fives in ten squares?

Two times five squares are how many squares?

What is one half of ten squares? Draw a line through one half of ten squares. Draw a line through another half of ten squares.

What is one half of ten books?

You have ten squares. Erase five of them. How many squares are left?

Ten squares less five squares are how many squares?

Ten squares less one half of ten squares are how many squares?

Ten fingers less one half of ten fingers are how many fingers? Can you show it?

Make five rows of circles with two circles in a row. How many circles have you made?

How many twos in ten circles?

Two circles and two circles and two circles and two circles and two circles are how many circles?

Five times two circles are how many circles?

What is one fifth of ten circles?

Draw a line through one fifth of ten circles. Draw a line through another fifth. Draw a line through one half of ten circles. If you should erase ten circles, how many circles would be left?

Ten fingers less ten fingers are how many fingers?

There are ten pupils in a class. One half of them are boys. How many boys are in the class? How many girls?

John has ten marbles in two pockets. There are five in one pocket. How many in the other pocket?

Ida had ten ducks and one fifth of them died. How many died? How many had she left?

Martha had ten hens. She sold one half of them to a neighbor and one fifth of them to her mother. How many did she sell? How many had she left?

There are ten apples on a plate. How many of us can take five apples apiece? How many of us can take two apples apiece?

How many cents must be added to five cents to make a dime?

Count by twos to ten.

Count by fives to ten.

How many pairs of horses in ten horses?

How many five-cent postage stamps can I buy with a dime?

Copy and fill blank spaces:

One half of 10 roses is —— roses.
5 turkeys and —— turkeys are 10 turkeys.
10 peaches less —— peaches are no peaches.
10 marbles less —— marbles are 5 marbles.
One fifth of 10 knives is —— knives.
10 cents will buy —— two-cent postage stamps.
10 cents will buy —— tops at 5 cents apiece.

LESSON 33.

ORAL EXERCISE.

(Make clear to the child the meaning of the words *sum* and *difference.*)

You will find in this lesson some *tables* of the sums and differences that we have learned. They are all the sums and differences of two numbers that we shall ever have when we use no number greater than ten. All the sums are found in Table 1. Count them and see how many there are. Yes, there are just twenty-five of them; and now we must learn them so that we shall remember them.

Can you tell me how many are 4 and 2? How many are 2 and 4? How many are 6 less 2? How many are 6 less 4?

How many are 5 and 3? Now tell the rest about it, just as you did about 4 and 2.

6 and 3 are how many? Tell the rest about it.

See these figures that I write on the board (writing—

$$\begin{array}{cccccc} 3 & 2 & 4 & 4 & 5 & 2 \\ \underline{2} & \underline{3} & \underline{3} & \underline{2} & \underline{2} & \underline{6}) \end{array}$$

See if you can tell me the sum of each pair of numbers I have expressed, *at once.*

See these figures that I write on the board (writing—

$$\begin{array}{cccccc} 7 & 5 & 6 & 8 & 9 & 9 \\ \underline{2} & \underline{3} & \underline{3} & \underline{2} & \underline{4} & \underline{2}) \end{array}$$

Tell me, at once, the difference of each pair of numbers I have expressed.

We shall take exercises like these every day until we have learned these tables thoroughly. Study the tables at your seat until you can answer such questions quickly.

NOTE.—A little time should be spent in this work every day, for a long while, whether the work is suggested in the oral exercises for each lesson or not. Take great care that the pupil gives his answers from memory each time. *Do not allow him to "count" one number on to the other, or off from it.*

Practice should enable the pupil to give results *at sight,* where only two numbers are involved, as readily as he can read a word of two or three letters. It is well to have the combinations of two figures written on cards. Then show a card to the class for an instant, put it out of sight, and call for the result.

FOR STUDY.

Addition and Subtraction Tables.

TABLE 1.

1+1= 2
1+2= 3
1+3= 4
1+4= 5
1+5= 6
1+6= 7
1+7= 8
1+8= 9
1+9=10

2+2= 4
2+3= 5
2+4= 6
2+5= 7
2+6= 8
2+7= 9
2+8=10

3+3= 6
3+4= 7
3+5= 8
3+6= 9
3+7=10

4+4= 8
4+5= 9
4+6=10

5+5=10

TABLE 2—SUMS.

2	1	3	1	4	1	5	1	6	1	7	1	8	1	9	1
1	2	1	3	1	4	1	5	1	6	1	7	1	8	1	9

2	3	2	4	2	5	2	6	2	7	2	8	2	3	4	3
2	2	3	2	4	2	5	2	6	2	7	2	8	3	3	4

5	3	6	3	7	3	4	5	4	6	4	5
3	5	3	6	3	7	4	4	5	4	6	5

TABLE 3—DIFFERENCES.

2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10	10
1	1	2	1	3	1	4	1	5	1	6	1	7	1	8	1	9

4	5	5	6	6	7	7	8	8	9	9	10	10	6	7	7	8
2	2	3	2	4	2	5	2	6	2	7	2	8	3	4	3	5

8	9	9	10	10	8	9	9	10	10	10
3	3	6	3	7	4	4	5	4	6	5

SIGHT ADDITION.

Give results at once from memory.

6	8	6	5	4	2	5	3	7	4	8	2	9	3	5	4	5
4	2	3	2	3	6	4	7	2	4	1	4	1	6	3	6	5

6	3	8	3	7	4	2
2	4	2	5	3	5	5

SIGHT SUBTRACTION.

Give results from memory quickly.

9	8	10	6	9	10	7	8	5	10	9	7	7	10	9	8	4
2	3	7	2	4	2	4	2	3	6	3	1	2	3	5	1	2

9	9	6	10	8	9	10
9	4	4	5	4	1	4

LESSON 34.

ORAL EXERCISE.

Each of you may count out 11 splints. Now put a rubber band around 10 of them. How many are loose? How many bunches of 10 can you make out of 11 splints? How many are left over? Then 11 is 10 and —?

Now each of you count out 9 splints. Lay them down and count out 2 more and put with them. How many have you in all? —— what?

How many splints are 9 splints and 2 splints?

How many are 9 and 2? 2 and 9? 11 less 2? 11 less 9?

How many are 8 and 3? 3 and 8? 11 less 3? 11 less 8?

(Teach the four statements for 7 and 4, 6 and 5.)

Harry may tell a story about 8 and 3, using beans to illustrate.

Margaret may tell a story about 11 less 8, using buttons to illustrate.

Johnnie has a nickel. How many more cents must he get to buy a book that costs 11 cents? 5 cents and how many more make 11 cents? etc.

Florence had a family of 11 dolls. She gave 3 to her little sister and 2 to her cousin. How many had she left? —— what?

Quick answers:

6 and 5 are how many?
7 and 4?
3 and 8?
11 less 5?
9 and 2?
5 and 6?
8 and 2?
10 and 1?
3 and 7?

How many with 8 make 10?
With 5? With 7? With 3? etc.

Give these sums at sight (writing on the board —

$$\begin{array}{cccccc}3 & 4 & 6 & 4 & 3 & 5\\ \underline{7} & \underline{5} & \underline{3} & \underline{2} & \underline{2} & \underline{2}\end{array}\ \text{etc.)}$$

Give these differences at sight (writing —

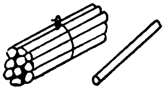
$$\begin{array}{rrrrr}10 & 8 & 5 & 9 & 8\\ \underline{3} & \underline{2} & \underline{4} & \underline{6} & \underline{4}\end{array}\ \text{etc.)}$$

What did we do with the ten splints? How many bunches did we have? How many splints in the bunch? Take eleven splints. Put a rubber band around ten of them. How many loose splints are there? Lay the splints down on the table in this way. (Take care to lay the single splint just at the right of the bunch.) How many of this kind have we? (Pointing to the bunch.) How many of this kind? (Pointing to the single stick.) What figure will tell the number of bunches? (1) What figure will tell the number of loose sticks? (1) Charlie, go and make these figures nicely on the board, taking care to put the figures down just as the sticks are lying. Now, which figure stands for the bunch? Point to it. Which figure stands for the loose stick? Point to it. Which figure do you think is worth more than the other? Why do you think so? Is the figure that stands for the bunch on the left-hand side or the right-hand side of the other?

Take ten splints. Put a band around them. How many bunches do ten sticks make? Are there any loose sticks now? How many figures did we use to write ten? Make them on the board. What does the one stand for? The other figure, looking like an O, is called *zero*. On which side of the 1 did we write it? Have we any loose sticks with the bunch when the number is 10? What stands in the place where we put figures that stand for loose sticks? *You may remember that we always put zero in that place when we write figures for one or more bunches, but have no loose sticks to write a figure for.*

NOTE.—The teacher will now see why we have said that splints, or sticks, are the most convenient counters for teaching the elements of number to young children. We know of no other way in which a child can be led to see the use of our decimal system so well as by bundling his ten sticks and putting a band around them. He now has ocular evidence that he has a new kind of one which actually contains ten of the former kind. He may understand that a dime is worth ten cents; but he does not *see* the ten cents in the dime. With our common numeral frame, he may see the ten balls on one wire; but he does not see the ten bound together as one of a new kind.

By carefully laying his bundles and his loose sticks in their proper order, and then making the figures for them respectively, in relations exactly corresponding, he is led naturally and intelligently to see how figures take on a place value, or "local" value, as it is commonly called in the books. Furthermore, by this plan, carefully followed for a time, he will come to see that *moving a figure one place to the left* multiplies its value by ten—not by "the annexing of a zero," as he is so often told *falsely*.



11 FOR STUDY. **Eleven**

How many splints are there in the picture? How many in the bundle? How many loose?

How many splints are 10 splints and 1 splint?

Make 9 straight lines in a row. Make 2 more beneath them. How many lines have you made all together?

How many are 9 lines and 2 lines?

How many are 2 lines and 9 lines? —— what?

Cover 2 lines. How many can you see?

Cover 9 lines. How many can you see?

How many are 11 lines less 2 lines?

How many are 11 lines less 9 lines?

Make 2 squares near together. Make a triangle just beneath them. How many sides have 2 squares?

How many sides has a triangle?

How many are 8 sides and 3 sides?

How many are 3 sides and 8 sides? —— what?

Cover 3 sides. How many sides can you see?

Cover 8 sides. How many sides can you see?

How many are 11 sides less 3 sides? —— what?

How many are 11 sides less 8 sides? —— what?

Make 7 crosses in a row. Beneath them make 4 crosses. How many crosses have you made?

How many crosses are 4 crosses and 7 crosses?

Cover 4 crosses. How many can you see?

Cover 7 crosses. How many can you see?

How many crosses are 11 crosses less 7 crosses?

Make 2 squares joined together. In the first square make six dots in 2 rows. In the other make 5 dots, 1 in the center and 1 near each corner. How many dots are there in both squares?

How many are 6 dots and 5 dots? —— what?

How many are 5 dots and 6 dots? —— what?

Cover 5 dots. How many can you see?

How many are 11 dots less 5 dots?

Erase 6 dots. How many are 11 dots less 6 dots?

John saw 4 crows on the ground and 7 flying about. How many crows did he see? How many crows are 4 crows and 7 crows?

A hen had 11 chickens. A hawk caught 3. How many were left?

A blind beggar met 2 boys. One boy gave him 5 cents and the other gave him 6 cents? How many cents did both boys give him?

A lady had 11 turkeys. A fox carried off 2 of them. How many were left?

Frank bought 6 apples from an Italian for 7 cents and sold them for 11 cents. How many cents did he gain?

I have 11 letters to mail and I have only 3 stamps. How many more stamps must I buy?

Grace has 11 cents. There is 1 five-cent piece and the rest are one-cent pieces. How many one-cent pieces has she?

How many crosses are 11 crosses less 4 crosses?

Cover 11 crosses. How many can you see?

Copy and fill blank spaces:

8 apples−3 apples= —— apples.
11 bells−5 bells= —— bells.
4 sheep+ —— sheep=11 sheep.
1 door+ —— doors=11 doors.
11 trees−8 trees= —— trees.
2 cakes+ —— cakes=11 cakes.
6 pencils+ —— pencils=11 pencils.
11 ducks−7 ducks= —— ducks.

$2+4+3=$
$5+6-2=$
$4+7-5=$
$9-6+8=$
$11-7+5=$
$1+10-8=$

LESSON 35.

ORAL EXERCISE.

Count out 12 splints. Put a band around 10 of them. How many loose ones have you? Then 12 is 10 and —?

Lay down your bunch of splints with the two loose splints just at the right of them. Mary, make a figure for the bunch, on the board. John, put a figure for the loose sticks in the right place near Mary's 1. Where are the loose sticks? Where is the figure for them? Where is the bunch? Where is the figure for it?

(Teach 9 and 3, 8 and 4, 7 and 5, 6 and 6, 2 times 6, one half of 12, and how many 6's in 12.)

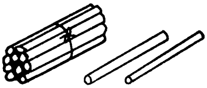
Mary wants to buy a dish that costs 12 cents. She has a dime. How many more cents must she have to buy the dish? Charles may tell a story about 7 and 5, using splints. Katie may tell a story about 12 less 3, using beans, etc.

All of you go to the board. Write the figures for 12, neatly. Write a word after them that says that this 12 means peaches.

Quick answers:

9 and 3 are how many? 6 and 6? 12 less 5? 9 less 4? etc.

(Practice for three minutes in "sight addition" and "sight subtraction," reviewing the tables.)



12 FOR STUDY. Twelve

How many splints are there in the bundle?
How many loose splints are there?
How many splints are there in the picture?
How many splints are 10 splints and 2 splints?
How many are 2 splints and 10 splints?
How many are 12 splints less 2 splints?
How many are 12 splints less 10 splints?

Make 3 triangles in a row. Beneath them make 1 more. How many sides have 3 triangles? How many sides has 1 triangle?

How many sides have 3 triangles and 1 triangle together?

How many are 9 sides of triangles and 3 sides?
How many are 3 sides of triangles and 9 sides?
How many are 12 sides less 9 sides? —— what?

Make 2 squares. Beneath them make 1 square. How many sides have 2 squares? How many sides has 1 square?

How many sides have 3 squares?
How many are 8 sides of squares and 4 sides?
How many are 4 sides and 8 sides?
How many are 12 sides less 4 sides?
How many are 12 sides less 8 sides?

Write the word *ceiling* on your slate. Beneath it write the word *house.* How many letters are there in the word ceiling? How many letters in the word house? How many letters in both words?

How many are 7 letters and 5 letters?

How many are 5 letters and 7 letters?

Cover 5 letters. How many letters are left uncovered?

How many are 12 letters less 5 letters?

Erase 7 letters. How many are 12 letters less 7 letters?

Make 6 circles in a row. Beneath them make 6 more circles in a row. How many circles have you made all together?

How many circles are 6 circles and 6 circles?

How many times 6 circles in 12 circles?

What is one half of 12 circles?

2 times 6 circles are how many circles?

Erase 6 circles. How many are 12 circles less 6 circles?

How many trees are there in an orchard that has 8 apple trees and 4 pear trees?

How many are 4 apples and 8 apples?

How many cows has a milkman who has 7 Jersey cows and 5 Durham cows?

From a board 12 feet long a carpenter cut off 3 feet. How many feet were left?

In a flock of 12 ducks and geese, 6 of them were ducks. How many geese were there?

Blanche has a dime. How many more cents must she get to have 12 cents?

Walter had 12 cents and paid 5 for car-fare. How many cents had he left?

There are 12 animals in the pasture. 4 of them are horses and the rest of them are cows. How many cows are there in the pasture?

Copy and fill blank spaces:

12 pictures−7 pictures= —— pictures.
9 cows+ —— cows=12 cows.
4 pens+8 pens= —— pens.
7 geese+ —— geese=12 geese.
12 eggs−10 eggs= —— eggs.
8 pies+ —— pies=12 pies.
6 hours+ —— hours=12 hours.

7+5−1=
9+3−4=
1+11−3=

12−5+4=
6+6−9=
2+10−7=

12−8+3=
2+8−7=
4+8−6=

LESSON 36.

ORAL EXERCISE.

All go to the board. Write eleven neatly. Just below this write twelve. What figures in your writing stand for tens? Point to them. What figures stand for ones? Point to them. Write a word after 11 to show that 11 cats are meant. Write a word by the 12 to show that 12 cents are meant. Each may count out 12 splints. Now see how many piles of six splints each you can make out of 12. What, then, is one half of 12 splints? See how many piles of 2 splints each you can make. What is one sixth of 12 splints? Now see how many piles of 4 splints you can make. What is one third of 12 splints? How many piles of 3 splints can you make? What is one fourth of 12 splints?

Which is more, 6 times 2 splints or 2 times 6 splints? Prove it to me with the splints.

Which is more, 3 times 4 splints or 4 times 3 splints? Prove it. Go to the board. Make a nice row of 4 dots. Make two more such rows beneath them. How many rows of 4 dots have you? How many dots in all? How many dots in a row if you look up and down? How many such rows? Can you show me, then, by your dots, that 3 times 4 dots is just the same as 4 times 3 dots? Who can do it?

Oranges cost 4 cents each. How many oranges can I buy for 12 cents? Show me with splints.

How many two-cent postage stamps can I buy for 12 cents? Show me.

(Review the "Tables.")

Quick answers:

One half of 12 are how many?
One third of 12? etc.

9 and 3?
12 less 7? etc.

FOR STUDY.

Make 12 crosses in a row. Count one half of them and draw a line between them and the next half.

How many crosses are there in a half of 12 crosses?

Draw a line through 2 crosses. Draw a line through as many more 2's as you can. Through how many 2's have you drawn a line?

What is one sixth of 12 crosses?

Make 12 circles in a row. Draw a line through 4 of them. Draw a line through as many more 4's as you can. Through how many 4's have you drawn lines?

3 times 4 circles are how many circles?

What is one third of 12 circles?

What is one third of 12 eggs?

Make 12 stars in a row. Draw a line through 3 of them. Draw a line through as many more 3's as you can. Through how many 3's have you drawn lines?

How many 3's in 12 stars? What kind?

4 times 3 stars are how many stars?

What is one fourth of 12 stars?

A cook had 12 eggs and used one fourth of them for a pudding. How many eggs did she use? How many were left?

Harry has 12 buckets of water to carry for his mother. If he carries 2 buckets at a time, how many times must he go to the well?

3 squares have how many more sides than 3 triangles? How many more has 1 square?

Roy picked one third of 12 quarts of berries in the forenoon and one half of 12 quarts in the afternoon. How many quarts did he pick in all?

How many feet have 3 horses?

How many feet have 6 hens?

How many wheels have 3 tricycles?

In an orchard are 12 trees. One half of them are apple trees, one third of them are pear trees, and one sixth of them are cherry trees. How many trees are there of each kind?

A boy bought 12 eggs. He fell and broke one sixth of them. How many eggs did he break? How many eggs were not broken?

Copy and fill blank spaces:

One half of 12 oranges is —— oranges.
One sixth of 12 pears is —— pears.
12 wheels will supply —— wagons.
In 12 skates are —— pairs of skates.
One third of 12 pencils is —— pencils.
One fourth of 12 desks is —— desks.

$3+9-4=$ $\quad$ $4+8-2=$ $\quad$ $1+11-9=$
$5+7-6=$ $\quad$ $2+10-6=$ $\quad$ $12-5+3=$
$12-10+9=$ $\quad$ $12-9+8=$ $\quad$ $7+5-8=$

LESSON 37.

ORAL EXERCISE.

NOTE.—Be sure that every child has a foot-rule, with inches marked and numbered, the half and quarter inches marked also. It is better to have them neatly made of light wood, if such can be got. They cost but little, and often booksellers, merchants, and druggists will give them away merely to advertise their business. But if nothing better can be done, get some strips of stiff cardboard and *make one for each child.* Some of the older pupils will generally be glad to help in making them.

Explain now to the class that the sign =, which they have read *are,* may be read "equals" or "equal to."

Who knows the length of this measure (showing a foot-rule)? For what are such measures used? Who use them? Who has seen them used? Who could use one here, in the schoolroom? George, where could you use one here? (George might say, "I could use one to see how long that table is.") Well, you *may* measure the length of the table. Did he do his work well (addressing the class)? Mary, where would you like to use your rule? (Mary might say, "I'd like to find how high the table is.") Mary may find what *she* wants to know about the table.

(Have the children measure the blackboards, the desks, doors, windows, etc. Get them so interested in using the rule that they will want to measure everything in sight. Let them take their rules out occasionally and measure things out of doors.)

Your foot-rules are marked off into shorter lengths. Who knows what one of these is called? (An inch.) Clarence may show me an inch on his rule. Laura may show me an inch on her rule. How many inches are marked on the rule? How many inches are there in a foot? I will write it on the board (writing—

"12 inches make 1 foot,"
and "1 foot=12 inches.")

Who knows when the inch is used for measuring? Who could use it on something in the schoolroom? Grant, where would you use it? (Grant might say, "I would use it to find how long my book is.") Each of you may find how long his book is. How wide is it? How wide are your slates? How long is your middle finger? How wide is a board in the floor?

How many inches in a foot? How many inches in one half a foot? Show me on your rule. 4 inches is what part of a foot? What is one fourth of 12 inches? Show me on your rule. What part of a foot is 3 inches? What is one sixth of 12 inches? What is one sixth of a foot? Show me one sixth of a foot on your rule.

Who knows what this measure is (showing a yardstick)? Who use measures like this? What are they used for? Emma may put her foot-rule on one end of this yard-rule. Ira may put his rule against the end of Emma's. Hattie may put her rule against Ira's. How many foot-rules on the yard-rule? How many feet make a yard? I will write on the board beneath the "12 inches make 1 foot," "3 feet make 1 yard," and "1 yard=3 feet." What is one third of a yard? Show me one third of a yard on this yardstick. All go to the board. Each make a line you think is a foot long. Measure it carefully and see if you are right. Make a line half a foot long. Measure it, etc.

***Quick answers*:**

3+3=?
10−7=?
3+9=?
12−3=?
6+5=?
8−2=?
6+4=?
8+3=?
3+8=?
11−8=?
11−3=?
6+6=?
10−8=?
Etc.

FOR STUDY.

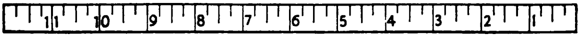


TABLE.

12 inches make 1 foot. 1 foot=12 inches.
3 feet make 1 yard. 1 yard=3 feet.

Draw a line one half of a foot long on your slate. Measure it carefully with your rule. How many inches long is this line?

How many inches are there in one half of a foot?

How many times 6 inches are there in 12 inches?

Draw a line one third of a foot long. How many inches long is this line?

How many inches are there in one third of a foot? How many more inches are there in one half of a foot than in one third of a foot?

How many times 4 inches are there in 12 inches?

Draw a line one fourth of a foot long. How many inches are there in this line?

How many inches are there in one fourth of a foot?

How many times 3 inches are there in 12 inches?

Which is longer, one third of a foot or one fourth of a foot? How many inches longer?

How many inches longer is one half of a foot than one fourth of a foot?

Draw a line one sixth of a foot long. How many inches are there in this line?

How many inches are there in one sixth of a foot?

How many times 2 inches are there in 12 inches?

How many more inches are there in one third of a foot than in one sixth of a foot? How many times as many?

How many more inches are there in one fourth of a foot than in one sixth of a foot?

Draw a line 1 yard long. How many feet in this line? How many feet in 1 yard?

How many feet are there in one third of a yard?

How many feet are there in 2 yards? In 3 yards? In 4 yards?

Our carpet is a yard wide. How many feet wide is it?

There are 6 yards of ribbon in one roll and 5 yards in another roll. How many yards are there in both rolls?

The top of the door is 7 feet from the floor. How many feet higher must the ceiling be to be 10 feet from the floor?

There are 7 feet of tail on Sam's kite, but he wants 12 feet. How many more feet must he put on?

Hattie wants 1 foot of lace. She has made 8 inches. How many more inches must she make?

Copy and fill blank spaces:

12 inches less 10 inches are —— inches.
3 inches and —— inches are 1 foot.
1 foot and —— feet are 1 yard.
One half of a foot is —— inches.
One sixth of a foot is —— inches.
One fourth of a foot is —— inches.
One third of a foot is —— inches.
In 3 feet are —— yard.
In 9 feet are —— yards.
In 4 yards are —— feet.
In 6 feet are —— yards.

$5+7-3=$
$9-5+8=$
$12-6+4=$
$11-3+2=$

$2+10-9=$
$8+3-6=$
$12-8+7=$
$10-5+3=$

$6+6-11=$
$3+7-5=$
$9+3-6=$
$7+5-9=$

LESSON 38.

ORAL EXERCISE.

Take 13 splints. Bunch 10 of them. What have you now? (1 bunch and 3 loose ones.) One bunch and 3 loose ones are how many ones? 10+3 are how many? Lay them on the table with the loose splints on the right-hand side of the bunch. Martha may make on the board the figures that mean thirteen. Where is the bunch? Where is the figure that stands for the bunch? Who can read what is on the board?

Notice how I write the word thirteen on the board. See how I divide it. (Write "thir-teen.") What word does the first part look like? (Three.) The last part? (Ten.) Well, that is what the word means, *three and ten.* So you see how easy it is to remember that 13 is 3 and 10, because the word says three and ten.

(Teach 9 and 4; 8 and 5; 7 and 6.)

Each child may take 7 splints. Lay them in the palm of your open hand. Shut your eyes. (Now let the teacher put 6 more splints in each child's hand.) Open your eyes. How many splints have you now? How many did I put into your hand? How many are 7 and 6? 6 and 7?

Bunch 10 of them. What have you now?

Frank may call his splints marbles, and tell a story about his gain.

Every one hold up 13 splints. Shut your eyes. (Now let the teacher take 7 splints from each.) Open your eyes. How many splints have you now? How many did I take away? How many are 13 less 7? 13 less 6? Fannie may call her splints chickens, and tell a story about her loss.

Take 11 splints. Bunch 10 of them. What have you? Lay them in right order (loose splint at the right-hand side of the bunch) on the table. Take 2 more splints. I want you to put them with the 11 splints. Where shall we put them? Shall they be put with the bunch or with the loose splints? You *may* put them with the loose splints. How many loose splints have you now? How many splints are there in the bunch and loose splints together? How many are 11 splints and 2 splints? 2 splints and 11 splints? Take 2 splints from 13 splints. From what did you take them, from the bunch or from the loose splints? How many *loose* splints are left? How many of the 13 splints are left? How many are 13 splints less 2 splints? When we want to put loose splints with a bunch and loose splints, with which shall we put them? When we want to take loose splints from a bunch and loose splints, from which shall we take the loose splints?

$1+2$ are how many? $11+2$ are how many? Show with splints.

$3-2$ are how many? $13-2$ are how many? Show with splints.

Do you disturb the bunch when you take 2 from 13?

Don't you see that it is like putting 2 with 1, or like taking 2 from 3, which we learned long ago?

(Practice rapidly on the combinations of Table 1, Lesson 33. Four forms for each combination.)

Short exercises in "sight" addition and "sight" subtraction.

Quick answers:

6 and 7 are how many?

$4+5=?$
$5+7=?$
$8-3=?$
$5+8=?$
$6+7=?$
$3+6=?$
$5+5=?$
$5+6=?$
$10-6=?$
$3+4=?$
$9+4=?$
$10+3=?$
$11-8=?$
$12-4=?$
$13-9=?$

FOR STUDY.

Make 3 triangles in a row. Beneath them make a square. How many sides have the triangles together? How many sides has the square?

How many sides are 9 sides and 4 sides?

How many are 4 sides and 9 sides?

Cover 4 sides. How many sides are left in sight?

How many are 13 sides less 4 sides?

Erase 9 sides. How many are 13 sides less 9 sides?

Make 8 dots in a line. Now make 5 more. How many are there? —— what?

How many dots are 8 dots and 5 dots?

How many dots are 5 dots and 8 dots?

How many are 13 dots less 5 dots?

Erase 8 dots. How many dots are left?

How many are 13 dots less 8 dots?

Write the word *sewing-machine* on your slate. How many letters on the left-hand side of the hyphen? How many on the right-hand side? How many in both parts of the word?

How many are 6 letters and 7 letters?

How many are 7 letters and 6 letters? —— what?

How many are 13 letters less 6 letters?

How many are 13 letters less 7 letters?

There were 13 roses on a bush before Mattie picked any. There were only 6 left after she picked some of them. How many did she pick?

There were in the room 9 chairs and 13 persons. How many persons had to stand?

May had 5 handkerchiefs and hemmed enough to make 13. How many did she hem?

A boy counted 13 insects. 4 were bees and the rest flies. How many flies were there?

A blind beggar met two boys. One gave him 6 cents and the other gave him enough to make 13 cents in all. How many cents did he give?

Copy and fill blank spaces:

13 less —— = 10.
13 oranges less 6 oranges are —— oranges.
8 jackknives and 5 jackknives are —— jackknives.
4 trees and —— trees are 13 trees.
7 lambs and —— lambs are 13 lambs.
13 kites less 5 kites are —— kites.
13 ladies less 9 ladies are —— ladies.

$4+9-6=$
$13-8+6=$
$4+5+4=$
$11-6+8=$

$10-3=$
$9+4=$
$3+4+5=$
$13-8+3=$

$5+7-9=$
$10-4+5=$
$7+6-3=$
$13-9+4=$

LESSON 39.

ORAL EXERCISE.

Take 14 splints. Bunch 10 of them. What have you now? How many are 10 splints and 4 splints? 4 and 10?

Lay your splints on the table in right order.

Edna may make the figures that stand for fourteen on the board. On which side of the ten are the ones? On which side of the figure that stands for the ten is the figure that stands for the ones? Where are the ones? Where is the figure that stands for the ones?

The word *fourteen* means *four* and *ten*.

(Teach 9 and 5; 8 and 6; 7 and 7; 7's in 14; $\frac{1}{2}$ of 14; 2's in 14; $\frac{1}{7}$ of 14.)

(Have some rapid work for three minutes in the Table of Sums; that is, Table 1. Let each combination be treated in its four forms, for example: 5+4, 4+5, 9—4, 9—5. Keep up the practice in "sight" addition and "sight" subtraction.)

Quick answers:

9+5=?
9—4=?
4+8=?
8—6=?

8+6=?
4+7=?
14—5=?
4+9=?

12—6=?
7+7=?
10—4=?
3+7=?

8+3=?
11—3=?
6+4=?
5+5=?

7+5=?
7+6=?
5+8=?
13—7=?

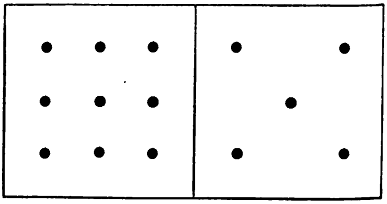
FOR STUDY.

1 bunch and 4 loose splints are how many? What kind?

How many are 4 books and 10 books?

How many are 14 bells less 4 bells? 14 bells less 10 bells?

Draw 2 squares with sides about an inch long. Make 9 dots in the first one and 5 dots in the second, in this way:



How many dots are there in both squares?

How many are 9 dots and 5 dots? How many are 5 dots and 9 dots?

Cover 5 dots. How many dots can you see?

How many are 14 dots less 5 dots?

Erase 9 dots. How many are 14 dots less 9 dots?

How many are 5 cows and 9 cows?

Make 2 squares. Beneath them make 2 triangles. How many sides have 2 squares? How many sides have 2 triangles?

How many are 8 sides and 6 sides? 6 sides and 8 sides? —— what?

Ned carried in 2 armfuls of wood. In the first were 6 sticks. How many must there be in the second armful to make 14 sticks in both?

Cover the 2 triangles. How many sides are left in sight?

How many are 14 sides less 6 sides?

Erase 2 squares. How many sides are left?

How many are 14 sides less 8 sides?

Make 7 stars in a row. Beneath them make another row of 7 stars. How many stars have you made?

How many are 7 stars and 7 stars? —— what?

How many times 7 stars are there in 14 stars?

2 times 7 stars are how many stars?

What is one half of 14 stars?

Draw a line through 2 stars. Draw a line through as many more 2's as you can. Through how many 2's have you drawn a line?

How many times 2 stars are there in 14 stars?

7 times 2 stars are how many stars?

What is one seventh of 14 stars?

Erase 7 stars. 14 stars less 7 stars are how many stars?

Erase 7 more stars. 14 stars less 14 stars are how many stars?

A book has 2 covers. How many covers have 7 books?

Count 14 by 2's.

Carrie paid 14 cents for a dozen bananas. At that price what would half a dozen cost?

James was sick 14 days. How many weeks was he sick?

Count 14 by 7's.

A fly has 6 legs and a spider enough more to make 14 legs. How many legs has a spider?

There were 14 dollars in my purse. How many were left after I paid 9 dollars for a coat?

In an orchard are 10 apple trees and enough pear trees to make 14 trees in the orchard. How many pear trees are there?

Copy and fill blank spaces:

10 splints and —— splints are 14 splints.
One half of 14 apples is —— apples.
14 ducks less 8 ducks are —— ducks.
5 lemons and 9 lemons are —— lemons.
7 desks and —— desks are 14 desks.
In 14 there are —— 2's.
4 hands and —— hands are 14 hands.
In 14 there are —— sevens.
One seventh of 14 lambs is —— lambs.

$6+8-7=$
$14-10+5=$
$9+5-8=$

$3+9-8=$
$2+9-5=$
$7+7-9=$

$6+7-8=$
$13-9+6=$
$14-7+2=$

LESSON 40.

ORAL EXERCISE.

Take 15 splints. Bunch 10 of them. What have you? How many are 10 splints and 5 splints? 5 and 10? How many are 15 less 5? 15 less 10?

Lay your splints on the table in the same order as before. Mattie may make the figures that stand for fifteen. Where is the ten? Where is the figure that stands for it?

Where are the ones? Where is the figure that stands for it?

(Teach $9+6=15$; $6+9=15$; $15-6=9$; $15-9=6$; $8+7=15$; $7+8=15$; $15-7=8$; and $15-8=7$, with stories of gain and loss. Teach 3 times $5=15$; $\frac{1}{3}$ of $15=5$; 5 times $3=15$; $\frac{1}{5}$ of $15=3$; $12+3=15$. See Lesson 39 for suggestions.)

The word *fifteen* means *five* and *ten*.

I bought 3 oranges for 15 cents. How many cents did I pay for each? What part of 15 cents?

Count by 3's to 15, and each time you count make the figure or figures which tell how far you have counted. In the same way count by 5's to 15.

John's overcoat cost 15 dollars. His hat cost one fifth as many dollars. How many dollars did his hat cost?

How many yards are there in 15 feet? Show me with splints.

Fannie had 5 roses, 5 lilies, and 5 pansies. How many flowers had she?

An orchard had 9 apple trees, and peach trees enough to make 15 trees in the orchard. How many peach trees are there in the orchard?

On a pond are 8 geese. On the bank are enough more to make 15 in all. How many geese are there on the shore?

Quick answers:

$8+7=?$
$9-4=?$
$7+8=?$
$4+9=?$
$7+5=?$
$14-7=?$
$6-3=?$
$6\times9=?$
$6+9=?$
$5+8=?$
$9+6=?$
$5\times7=?$
$7+6=?$
$11-5=?$
$3+4=?$
$10-8=?$
$5+3=?$
$11-3=?$
$15-5=?$
$15-6=?$

How many feet are there in a yard?

How many inches in a foot?

(Practice the tables and "sight" work.)

FOR STUDY.

A bunch and 5 loose splints are how many splints?

How many are 10 sheep and 5 sheep?

There are 15 pupils in one class. 5 of them are boys and the rest are girls. How many girls are there in the class?

Make 3 triangles in a row. Beneath them make 2 more. How many sides have 3 triangles? How many sides have 2 triangles?

How many are 9 sides and 6 sides? 6 sides and 9 sides?

There are 6 cows in the pasture. How many more cows must be put into the pasture to have 15 cows in the pasture?

Cover 2 triangles. How many sides of the triangles are left in sight?

How many are 15 sides less 6 sides?

Erase 3 triangles. How many sides are left?

How many are 15 sides of triangles less 9 sides?

There were 15 chairs in the room, but Lucy carried out 6 of them. How many were left?

Make in a row the letter *n* 8 times. Beneath them make 7 *n*'s. How many *n*'s have you made?

How many are 8 *n*'s and 7 *n*'s? How many are 7 *n*'s and 8 *n*'s? —— what?

Helen has read 8 pages of her new book and she has 7 more pages to read. How many pages are there in the book?

Cover 7 *n*'s. How many are left in sight?

How many are 15 *n*'s less 7 *n*'s?

Erase 8 *n*'s. How many are left?

How many are 15 *n*'s less 8 *n*'s? —— what?

Ray's writing book has 15 leaves. He has finished 8. How many more leaves has he to finish?

Make 5 circles in a row. Beneath them make another row of 5 circles, putting each circle beneath a circle in the first row. Make another row of 5 circles beneath the other 2 rows. How many circles have you made?

How many are 3 times 5 circles?

How many times 5 circles are there in 15 circles?

What is one third of 15 circles?

A row up and down the page is called a column. How many columns of circles have you made?

How many circles have you made in each column?

How many are 5 times 3 circles? —— what?

How many times 3 circles are there in 15 circles?

What is one fifth of 15 circles?

Anna learned 3 verses of a poem each day for 5 days. How many verses were there in the poem? What part of it did she learn each day?

A fruit dealer received 15 baskets of peaches. When he opened them he found that one fifth had spoiled. How many baskets had spoiled?

How many strokes will a clock make in striking 7 and 8?

There were 15 saucers on a table, but 6 of them had no cups. How many saucers had cups?

Clara had a dime. How many more cents must she get to have 15 cents?

Andrew ran 3 errands, receiving 5 cents for each. How many cents did he receive for all?

How many nickels are worth 15 cents?

What is one third of 15 cents? What is one fifth of 15 cents?

Thomas picked 7 quarts of berries in the forenoon. How many quarts must he pick in the afternoon to have 15 quarts?

Copy and fill blank spaces:

10 boys and 5 boys are —— boys.
15 clocks less 6 clocks are —— clocks.
7 tables and —— tables are 15 tables.
One third of 15 lemons is —— lemons.
12 balls and one fourth of 12 balls are —— balls.
9 pears and —— pears are 15 pears.
One fifth of 15 lessons is —— lessons.
There are —— 3's in 15.

$7+8-9=$
$6+3-5=$
$13-7+9=$

$8+4-7=$
$11+4-6=$
$14-7+8=$

$10-3+6=$
$9-6+12=$
$15-8+3=$

LESSON 41.

ORAL EXERCISE.

Take 16 splints. Bunch 10 of them. What have you now? Write figures for sixteen on the board.

(Teach $10+6=16$; $6+10=16$; $16-6=10$; $16-10=6$; $9+7=16$; $7+9=16$; $16-7=9$; $16-9=7$. Have stories. Teach $8+8=16$; $16-8=8$; 2 times $8=16$; $\frac{1}{2}$ of $16=8$; 8 times $2=16$; $\frac{1}{8}$ of $16=2$; 4 times $4=16$; $\frac{1}{4}$ of $16=4$. Teach how many 4's in 16. Show it with splints.)

The word *sixteen* means *six* and *ten*.

Quick answers:

$7+9=?$
$5+6=?$
$16-6=?$
$9+5=?$
$8+8=?$
$4+7=?$
$6+4=?$
$8+5=?$
$7+6=?$
$14-7=?$
$8+6=?$
$12-3=?$
$13-6=?$
$6+8=?$
$6+10=?$
$4+8=?$
$11-4=?$
$15-9=?$
$8+7=?$
$16-8=?$

How many inches in one foot? In one half a foot?

(Practice on tables and "sight" exercises.)

FOR STUDY.

A bunch and 6 loose splints are how many plints?

How many are 10 trees and 6 trees?

There are 16 animals in a pasture. 10 of them re cows, and the rest are horses. How many orses are there?

Make 2 squares an inch on each side. In the irst make 9 dots. In the other make 7 dots. ow many dots have you made in both?

How many are 7 dots and 9 dots?

How many are 16 dots less 7 dots?

Erase 9 dots. How many dots are left?

How many are 16 dots less 9 dots?

Harry caught 9 trout and Willie caught 7 rout. How many did they catch in all? ——what?

Make 8 stars in a row on your slate. Beneath hem make another row of 8 stars. How many tars have you made?

How many are 8 stars and 8 stars?

How many are 2 times 8 stars?

How many times 8 stars are there in 16 stars?

What is one half of 16 stars?

Cover 8 stars. How many stars can you see?

How many are 16 stars less 8 stars?

Draw a line through 2 stars. Draw a line through as many more 2's as you can. Through how many 2's have you drawn a line?

How many times 2 stars are there in 16 stars?

How many are 8 times 2 stars?

What is one eighth of 16 stars?

How many are 16 stars less 16 stars?

Make 2 squares in a row. Beneath them make 2 more squares. How many sides has 1 square? How many sides have 4 squares?

How many are 4 times 4 sides?

How many times 4 sides are there in 16 sides?

What is one fourth of 16 sides?

Hattie had 16 apples on 2 plates. The same number of apples were on each plate. How many apples were there on each plate?

Count by 2's to 16, and at each count you make write on your slate the figure or figures which tell how far you have counted. In the same way count by 4's and 8's to 16.

A fruit dealer received 16 baskets of pears. One eighth were spoiled. How many baskets were spoiled?

A girl who earned 16 dollars a month, paid one half of it for board. How many dollars a month did she pay for board?

How many dollars shall I have left out of 16 dollars, if I pay 9 dollars for a coat?

At 4 cents a quart, how many quarts of milk can I buy for 16 cents?

A milkman has 9 cows. How many more must he buy to have 16 cows?

Copy and fill blank spaces:

10 dogs and —— dogs are 16 dogs.
7 hens and 9 hens are —— hens.
16 lambs less 8 lambs are —— lambs.
7 cups and —— cups are 13 cups.
6 lemons and —— lemons are 14 lemons.
6 lamps and —— lamps are 15 lamps.
9 rolls and —— rolls are 16 rolls.
5 miles and —— miles are 12 miles.
One half of 16 feet is —— feet.
One half of 14 boys is —— boys.
One third of 15 apples is —— apples.
One half of 12 inches is —— inches.
15 turkeys less 9 turkeys are —— turkeys.

2+9−3=
8+7−5=
3+4+6=
5+9−7=

16−8+ 6=
6+9−10=
11−6+ 9=
8+8− 9=

7+9−6=
5+8−6=
9+6−7=
12−6+5=

LESSON 42.

ORAL EXERCISE.

Take 17 splints. Bunch 10 of them. What have you now? Write the figures for seventeen. The word *seventeen* means *seven* and *ten.*

(Teach 10+7=17; 7+10=17; 17−7=10; 17−10=7; 9+8=17; 8+9=17; 17−8=9; 17−9=8. Stories.)

Take 18 splints. Bunch 10 of them. What have you now? Write the figures for eighteen. The word *eighteen* means *eight* and *ten.*

(Teach 10+8=18; 8+10=18; 18−8=10; 18−10=8; 9+9=18; 18−9=9; 2 times 9=18; $\frac{1}{2}$ of 18=9; 9 times 2=18; $\frac{1}{9}$ of 18=2; 3 times 6=18; $\frac{1}{3}$ of 18=6; 6 times 3=18; $\frac{1}{6}$ of 18=3.)

Quick answers:

9+9=?
9+8=?
5+9=?
14−8=?
4+7=?
7+7=?

13−6=?
6+8=?
6+6=?
9+7=?
8+3=?
7+6=?

15−10=?
9+ 6=?
17−10=?
10+ 7=?
8+ 5=?
8+ 7=?

7+8=?
8+9=?
16−8=?
7+4=?
18−9=?
11+7=?

FOR STUDY.

A bunch and 7 loose splints are how many splints? Write the figures for the number.

How many are 10 roses and 7 roses? 7 roses and 10 roses?

How many are 17 leaves less 7 leaves? 17 leaves less 10 leaves?

Make 3 triangles in a row. Beneath them make 2 squares. How many sides have 3 triangles? How many sides have 2 squares?

How many are 9 sides and 8 sides? How many are 8 sides and 9 sides?

Cover 8 sides. How many are 17 sides less 8 sides?

Erase 9 sides. How many are 17 sides less 9 sides?

Luther bought a dozen oranges for 9 cents and sold them for 17 cents. How many cents did he gain?

A bunch and 8 loose splints are how many? Write the figures for the number.

How many are 10 books and 8 books? 8 pens and 10 pens?

How many are 18 buckets less 8 buckets? 18 desks less 10 desks?

Make 9 circles in a row. Beneath them make another row of 9 circles. How many circles have you made?

How many are 9 circles and 9 circles?

How many are 2 times 9 circles?

How many times 9 circles are there in 18 circles?

What is one half of 18 circles?

Draw a line through 2 circles. Draw a line through as many more 2's as you can. Through how many 2's have you drawn lines?

How many times 2 circles are there in 18 circles?

How many are 9 times 2 circles?

How many is one ninth of 18 circles? What?

Erase 9 circles. How many are 18 circles less 9 circles?

Make 6 dots in a row. Beneath them make another row of 6 dots. Beneath them make still another row of 6 dots. How many dots have you made?

How many are 6 dots and 6 dots and 6 dots?

How many are 3 times 6 dots?

How many times 6 dots are there in 18 dots?

What is one third of 18 dots?

How many are 6 times 3 dots?

How many times 3 dots are there in 18 dots?

What is one sixth of 18 dots?

How many are 18 dots less 18 dots?

Count by 2's to 18, and each time you count 2 make the figure or figures which tell how far you have counted. In the same way count by 3's, 6's, and 9's to 18.

How many blades have 9 pairs of scissors?

If Ralph has 18 apples and eats 2 a day, how many days will they last him?

How many strokes will a clock make in striking 8 and 9?

Ethel's writing book has 18 pages. She has finished one half of them. How many pages has she finished?

A boy earns 18 dollars a month and pays out one ninth of them for car-fare. How many dollars a month does he pay for car-fare?

Alice put 18 flowers in 6 bouquets, putting the same number of flowers in each. How many flowers did she put in each bouquet?

She sold the bouquets at 3 cents apiece. How many cents did she get for them?

A tea-merchant bought 18 chests of tea. One sixth of it was found damaged. How many chests were not damaged?

Nathan has earned 9 cents. How many more cents must he earn in order to have 18 cents?

If Ernest had 9 marbles more he would have 17 marbles. How many marbles has he?

How many yards long is a piece of cloth that is 18 feet long?

Copy and fill blank spaces:

10 cows and —— cows are 18 cows.
9 houses and —— houses are 18 houses.
8 turkeys and —— turkeys are 17 turkeys.
One half of 18 dollars is —— dollars.
One third of 18 feet is —— feet.

There are —— 3's in 18.
One sixth of 18 cows is —— cows.
18 pansies less 9 pansies are —— pansies
17 roses less 9 roses are —— roses.
One ninth of 18 pears is —— pears.
In 18 there are —— 2's.
There are —— 6's in 18.

9+9−10=
9+8− 7=
5+9− 7=
8+7− 6=
6+8− 5=

4+5−3+7=
8+3−6+8=
4+4+4+4=
5+5+5−6=
5+7−4+8=

LESSON 43.

ORAL EXERCISE.

Each count out 19 splints. Put a band around 10 of them. What have you now? How many are 10 and 9? 9 and 10? 19 less 9? 19 less 10? Write the figures for nineteen on the board. Tell what each figure stands for. The word *nineteen* means what, do you think? Yes, it means *nine* and *ten*.

Put another loose splint with your nine loose ones. How many loose splints have you now? What do you do when you have ten loose splints? (Bunch them.) How many bunches have you now? Two bunches are how many splints in all? Make a figure on the board that tells how many bunches in twenty. Have you any loose splints? Then, have you any figure to stand for loose splints? How, then, shall we know that this 2 stands for bunches? (Probably some one, remembering how ten was written, will say, "Put a zero at the right of the 2." The teacher should get this thought before them, if no child expresses it; and she should emphasize the fact that, in this way we know that the 2 stands for bunches or tens. This is the important point—not that the zero shows that there are no loose sticks.) Where are the bunches? Where is the figure that stands for the two bunches? If a stranger should come in here, how would he know that the 2 means bunches, or tens, and not loose splints, or ones? Yes, the zero at the right of it shows that.

NOTE.—The use of zero in Arithmetic is to fill places that would be vacant between any figure and the decimal point; this is to show the place of the significant figures, and their consequent local value. Keep this general and far-reaching principle before pupils till they shall clearly understand the real purpose of the zero.

How many splints are there in 1 bunch? In 2 bunches? How many are 10 splints and 10 splints? —— what? How many are 2 times 10 splints? How many times 10 splints are there in 20 splints? Show me. What is one half of 20 splints? Show me. The word *twenty* means *two tens.*

Take the bands off the two bunches. Lay the twenty splints in piles, putting 2 splints in a pile. How many piles do 20 splints make? How many times 2 splints are there in 20 splints? How many are 10 times 2 splints? What is one tenth of 20 splints? Show me.

(Teach in a similar way: 4 fives in 20; 4 times 5 are 20; $\frac{1}{4}$ of 20 is 5; 5 fours in 20; $\frac{1}{5}$ of 20 is 4. Emphasize that the word twenty means two tens. Do not neglect practice on tables and "sight" work.)

Quick answers:

10+10=?
10+ 9=?
10+ 8=?
10+ 7=?
10+ 6=?
9+ 8=?

9+7=?
9+6=?
8+9=?
8+8=?
6+8=?
7+8=?

7+7=?
6+7=?
5+9=?
9+9=?
3+8=?
8+4=?

7+4=?
4+5=?
5+6=?
6+4=?
5+8=?
6+6=?

Take 19 splints once more, bunch 10 of them and lay the 9 loose ones at the right of the bunch. Now, write on the board the figures for the number you have. I want to take 5 splints away from these lying on the table, to find out how many will be left. Robert may write 5 under the 9 on the board, and draw a line underneath. Where are the figures that tell how many splints we have in all? Where are the splints? Where is the figure that tells how many we want to take away? Where are these 5 splints which we want to take away? (Some one ought to say that they are in, or among, the loose splints.) John may take them out and lay them apart. How many loose splints are left? Make the figure for these four on the board, under the other figures that stand for loose splints. What have we done with the bunch? (Nothing.) Then make the figure for it just at the left of the 4. How many splints are left? Where are the figures that stand for the fourteen? Where is the figure that stands for the 5 we took away? Where are those 5? Point to them. 5 from 19 leaves how many? 5 from 9 leaves how many? Have we learned that before? You really took the 5 from the 9, did you not? Did you disturb the ten, or the bunch?

NOTE.—By exercises such as these, the child should be led to see that, in such cases as this, he is not to find a new difference, but is merely finding the same difference that he found when he had no tens. He should also see that his work with figures represents, step by step, a work with things.

FOR STUDY.

1 bunch and 9 loose splints are how many splints?

Write the figures for the number nineteen.

How many are 10 robins and 9 robins? 9 nests and 10 nests?

How many are 19 hens less 10 hens? 19 nests less 9 nests?

Make 10 dots in a row. Beneath them make 10 dots more in a row. How many dots are there in both rows?

How many dots are 10 dots and 10 dots?

How many are 2 times 10 dots? —— what?

What is one half of 20 dots?

How many bunches are there in 20 splints?

How many times 10 splints in 20 splints? Write twenty in figures.

How many times 2 splints in 20 splints?

Draw a line through 2 dots. Draw lines through as many more 2's as you can. Through how many 2's have you drawn lines?

How many times 2 dots are there in 20 dots?

How many are 10 times 2 dots?

What is one tenth of 20 dots?

Erase 10 dots. How many are 20 dots less 10 dots?

Make 5 stars in a row. Beneath this row make 3 more rows of 5 stars each. How many rows of stars have you made?

How many stars have you made in 4 rows?

How many are 5 stars and 5 stars and 5 stars and 5 stars?

How many are 4 times 5 stars?

How many times 5 stars are there in 20 stars?

What is one fourth of 20 stars?

How many columns of stars are there?

How many stars in each column?

How many are 4 stars and 4 stars and 4 stars and 4 stars and 4 stars?

How many are 5 times 4 stars?

How many times 4 stars are there in 20 stars?

What is one fifth of 20 stars?

Count by 2's to 20, at each count making the figure or figures which tell how far you have counted. In the same way count by 4's, 5's, and 10's to 20.

At 5 dollars a barrel how many dollars will 4 barrels of flour cost?

I have 20 cents in 5-cent pieces. How many 5-cent pieces have I?

They put "Quaker Oats" up in 2-pound packages. How many packages will 20 pounds make?

At 10 cents a quire how many quires of paper can be bought for 20 cents?

A buggy has 4 wheels. How many buggies will 20 wheels supply?

Walter brought 20 nuts to school. He gave one fourth of them to Rufus. How many nuts did he give to Rufus? How many had he left?

On Saturday afternoon 20 children went skating. One half of them were girls. How many girls were in the company? How many boys?

John and Albert met a beggar as they came to school. John gave him one fourth of 20 cents and Albert gave him one fifth of 20 cents. Which gave him the more cents? How many more?

Take 7 from 19 and show the figures.

What does the word *nineteen* mean? What does the word *twenty* mean?

Copy and fill blank spaces:

10 pencils and —— pencils are 20 pencils.
9 pens and —— pens are 19 pens.
One half of 20 apples is —— apples.
One tenth of 20 yards is —— yards.
One fifth of 20 slates is —— slates.
One fourth of 20 quail is —— quail.
20 boys less 18 boys are —— boys.
19 girls less 10 girls are —— girls.

7+9−8=
6+9−7=
8+5−6=
9+9−8=

9+8−7=
4+9−7=
8+6−7=
9+6−5=

3+8− 6=
9+5− 6=
8+9−10=
8+8− 7=

LESSON 44.

ORAL EXERCISE.

NOTE.—In this lesson, Table 1 presents at one view all the sums that the children have studied since we left the number ten and the previous tables. There are twenty of these combinations, which, added to the twenty-five in the previous table, make up *all the combinations of two numbers that we ever shall need to make*. These forty-five combinations are absolutely all that the child needs to store in his memory for any addition he will ever have to make. Nor will he ever have to learn anything new for subtraction, for we have seen repeatedly that 7+6=13 implies also that not only does 6+7=13, but that 13−6=7 and 13−7=6. The addition and subtraction table is now complete, and by sufficient drill it must be *thoroughly fixed in the memory*.

We have just seen that in adding 2 to 11 we really added 2 to 1, and in taking 5 from 19 we took 5 from 9. In a similar way it may be shown that in any addition or subtraction, in the longest and most complicated work that the arithmetician will ever have to perform, he will need to use no result that is not found in these two tables. When they are so thoroughly fixed in memory that he can give sum or difference instantly at sight, when any two figures are so placed as to call for the sum or difference of the numbers for which they stand (as in the tables 2 and 3, and in the "sight" addition and subtraction), he has taken a long stride toward accuracy and rapidity in calculation by numbers. Hence the importance of continuous and daily drill in the mastery of these tables. Use no splints or other objects in these drills unless it shall appear that some child has forgotten how the tables were first derived, and must be turned back by way of review.

In all these drills insist upon the greatest possible rapidity short of confusion.

Impress upon the pupil that in arithmetic he has but few things to fix in his memory, but he must fix those few things *exactly*.

Put the tables and "sight" combinations on the board and drill vigorously on them for ten minutes. Give each statement in Table 1 the four familiar forms.

FOR STUDY.

Tables.

TABLE 1.

6+5=11	7+4=11	8+3=11	9+2=11
6+6=12	7+5=12	8+4=12	9+3=12
6+7=13	7+7=14	8+5=13	9+4=13
6+8=14	7+8=15	8+8=16	9+5=14
6+9=15	7+9=16	8+9=17	9+9=18

TABLE 2—SUMS.

5	6	6	7	6	8	6	9	6	4	7	5	7	7	8	7	9	7
6	5	6	6	7	6	8	6	9	7	4	7	5	7	7	8	7	9

3	8	4	8	5	8	8	8	9	2	9	3	9	4	9	5	9	9
8	3	8	4	8	5	8	9	8	9	2	9	3	9	4	9	5	9

TABLE 3—DIFFERENCES.

11	11	12	13	13	14	14	15	15	11	11	12
5	6	6	6	7	6	8	6	9	4	7	5

12	14	15	15	16	16	11	11	12	12	13	13
7	7	7	8	7	9	3	8	4	8	5	8

16	17	17	11	11	12	12	13	13	14	14	18
8	8	9	2	9	3	9	4	9	5	9	9

"SIGHT" ADDITION.

Give results at once from memory.

5	6	8	9	4	7	9	8	5	8	9	9	8	6	6
9	7	5	6	7	5	4	9	6	7	5	8	4	5	9

9	6	7	9	5	7	8	4	5	7	4	5	7	9	8
2	8	9	3	8	4	3	9	7	8	8	5	6	7	6

"SIGHT" SUBTRACTION.

Give results at once from memory.

13	11	17	12	15	13	14	18	15	13
6	9	8	5	7	9	7	9	6	5

11	14	16	11	14	16	12	13	17	12
8	6	7	4	5	8	4	7	9	9

16	12	14	13	11	15	12	14	11	13
9	6	9	8	7	9	7	8	5	4

How many are 8+7? 7+8? 15 less 7? 15 less 8? Answer four similar questions for each combination in Table 1.

Learn Table 1 thoroughly. Then you can give *all* the results in Tables 2 and 3 and in the "sight" addition and "sight" subtraction, at once, from memory.

LESSON 45.

ORAL EXERCISE.

NOTE.—Give a short, rapid review on writing the numbers from 11 to 19.

Take 20 splints. Bunch them by tens. What have you?

Emma may make the figure which stands for the bunches. Why should a zero be put at the right of the 2? (To show that the 2 stands for tens, not for ones.)

Put a loose splint with the 2 bunches. What number have you? Lay them on the table in order. Ruth may make on the board the figure which stands for the 2 bunches. Now make the figure which stands for the loose splint. Where are the bunches? Where is the figure that stands for them? Where is the loose splint? Where is the figure that stands for it? Read what is written on the board. Write the word twenty-one.

Put another splint with the loose splint. What have you? Lay them in order. Oliver may make the figures which stand for 2 bunches and 2 loose splints. Where are the bunches? Where is the figure that stands for them? Where are the loose splints? Where is the figure that stands for them? Read what is written on the board.

(Teach 23, 24, 25, 26, 27, 28, 29, in a similar way. Let the children put another splint with the 9 loose ones. They will understand that a band should be put around the new ten. Let it be put on.)

How many bunches have you now? 10 splints and 10 splints and 10 splints are how many splints? 3 times 10 splints are how many splints? How many times 10 splints are there in thirty

splints? How many times can 10 splints be taken from 30 splints? What will be left? What is one third of 30 splints?

George may make on the board the figure that stands for 3 bunches. Now make the figure that shows that the 3 stands for bunches, not for ones. Where are the bunches? Where is the figure that stands for them? Read what you have written.

Quick answers:

$9+7=?$
$20+3=?$
$7+8=?$
$11-3=?$
$9+8=?$
$14-8=?$
$15-7=?$
$20+5=?$
$19-9=?$
$5+4=?$
$8+5=?$
$13-4=?$
$20+7=?$
$18-9=?$
$20-9=?$
$20+9=?$
$16-9=?$
$6+9=?$
$17-10=?$
$5+6=?$
$9+2=?$
$20+6=?$
$7+7=?$
$8+8=?$
$7+6=?$
$9+3=?$
$4+8=?$
$4+7=?$
$8+9=?$
$20-1=?$

Write figures for all the numbers from 20 to 30.

(Practice on tables and "sight" work.)

NOTE.—Take great care that in writing the compound names of numbers, as twenty-one, twenty-nine, etc., the hyphen is properly used.

FOR STUDY.

Make 10 squares in a row on your slate. Beneath them make another row of 10 squares. How many squares are 10 squares and 10 squares?

Make the figures which tell how many squares you have made. What does the word *twenty* mean?

Make 1 square beneath the 2 rows of 10 squares. How many squares have you now? How many are 20 squares and 1 square? Beneath the figures for 20 squares make the figures which stand for twenty-one squares.

Make another square beneath the 20 squares. How many are 20 squares and 2 squares?

Beneath the figures that stand for twenty-one squares, make the figures that stand for twenty-two squares.

Make another square beneath the 20 squares. How many are 20 squares and 3 squares?

Beneath the figures which stand for twenty-two squares, make the figures which stand for twenty-three squares.

Make another square with the 3 squares. How many are 20 squares and 4 squares?

Make the figures that stand for twenty-four squares.

Make another square with the 4 squares. How many squares have you? 20 squares and 5 squares are how many squares?

Make the figures which stand for twenty-five squares.

Make another square with the 5 squares. How many are 20 squares and 6 squares?

Make the figures that stand for twenty-six squares.

Make another square with the 6 squares. How many are 20 squares and 7 squares?

Make the figures that stand for twenty-seven squares.

Make another square with the 7 squares. How many are 20 squares and 8 squares?

Make the figures for twenty-eight squares.

Make another square with the 8 squares. How many are 20 squares and 9 squares?

Make the figures which stand for twenty-nine squares.

Make another square with the 9 squares. How many rows of 10 squares each have you made? 10 squares and 10 squares and 10 squares are how many squares? How many are 3 times 10 squares?

What does the word *thirty* mean? How many times 10 squares are there in 30 squares? How many 10's can be taken from 30? What is one third of 30 squares?

Make the figures that stand for thirty. What does the 3 stand for? How do you know? Why do you use the zero?

Copy these words, and write after them the figures that stand for the numbers they name:

twenty-two
twenty
twenty-eight
twenty-five
twenty-nine
twenty-four
nineteen
twenty-seven
eighteen
twenty-six
twenty-one
thirty
seventeen
twenty-three

Count from 12 to 30, and make the figures for the numbers as you count.

Copy and fill blank spaces:

2 times 10 apples are —— apples.
20 plums and 3 plums are —— plums.
20 hens and 5 hens are —— hens.
20 pens and 6 pens are —— pens.
20 ducks and 7 ducks are —— ducks.
20 slates and 8 slates are —— slates.
20 boys and 9 boys are —— boys.
3 times 10 desks are —— desks.

4+ 6+5=
7+ 8−6=
9+ 7−8=
6+ 8−7=
3+ 9−4=
9+ 8−7=
19−10+5=
7+ 4−6=
4+ 5−6=
2+ 9−8=
3+ 7−5=
4+ 9−6=
14− 6+8=
17−10+5=
8+5−7=
6+5−4=
3+4+9=
16−7−4=
18−9−6=
15−8−3=
3+4+8=

LESSON 46.

ORAL EXERCISE.

NOTE.—Review the numbers from 20 to 30 by holding up bunches and loose splints, and requiring the pupil to make on the board the figures which tell how many are held up.

Take 3 bunches of splints. Put 4 loose ones beside them. What have you? How many splints are 30 splints and 4 splints? Lay them on the table in order. Fannie may make the figures that stand for 3 bunches and 4 splints. Where are the bunches? Where is the figure that stands for them? Where are the loose splints? Where is the figure that stands for them? Read what you have written. Write the word for this number.

Bunch another ten. Who knows how many splints there are in 4 tens of splints? Frank may make the figure that stands for 4 tens. Now make the figure that shows that this 4 stands for tens. What is the name of the figure? Where are the 4 tens? Where is the figure for 4 tens? Read what you have written. The word *forty* means *four tens.*

Put 6 loose splints with the 4 bunches. Who knows how many splints there are in 4 bunches and 6 loose splints? Hattie may make the figures that stand for 4 bunches and 6 loose splints. Where are the bunches? Where is the figure for them? Where are the loose splints? Where is the figure for them? Read what you have written. Write the word forty-six.

Bunch another ten. Who knows how many splints there are in 5 tens of splints? Lucy may make the figure which stands for 5 tens. Why should you write a zero at the right of the 5? (To show that the 5 stands for tens.) Read what you have written. The word *fifty* means *five tens.*

Put 7 loose splints with the 5 bunches. Who knows how many splints there are in 5 bunches and 7 loose splints? Harry may make the figures which tell how many splints there are in 5 bunches and 7 loose splints.

(In a similar way teach 60, 68, 70, 73, 80, 84, 90, 99, or other numbers up to 99.)

Show me 37 splints (by bunches and loose splints); 46 splints; 72 splints; 80 splints; 96 splints, etc.

How many are 8 tens? 6 tens? 4 tens? 5 tens? 9 tens? 7 tens?

John bought 2 pounds of dates at 10 cents a pound. How many cents did he pay for them?

Alice bought 5 yards of ribbon at 10 cents a yard. How many cents did the ribbon cost?

There are 9 rows of trees in an orchard and 10 trees in a row. How many trees are there in the orchard?

Quick answers:

9+5=?
8+7=?
6+8=?
5+6=?
8+4=?
18+4=?
28+4=?
38+4=?
48+5=?
4+9=?
7+5=?
6+7=?
5+8=?
58+5=?
68+5=?
78+5=?
88+5=?
7+9=?
13−7=?
14−8=?
15−9=?
7−4=?
17−4=?
27−4=?
37−4=?
47−4=?
57−4=?
67−4=?
77−4=?
87−4=?
16−9=?
17−8=?
18−9=?
19−9=?
77−5=?
87−5=?
97−5=?
15−7=?
25−7=?
35−7=?

FOR STUDY.

Make 10 stars in a row on your slate. Beneath the row make 2 more rows of 10 stars each. How many stars are there in 3 rows?

Make the figures for thirty.

Make 1 star beneath the 3 rows.

How many are 30 stars and 1 star? 30 stars and 2 stars? 30 stars and 3 stars? 30 stars and 4 stars? 30 stars and 5 stars? 30 stars and 6 stars? 30 stars and 7 stars? 30 stars and 8 stars? 30 stars and 9 stars?

Make the figures for thirty-one beneath the figures for thirty. Beneath them make the figures for thirty-two; thirty-three; thirty-four; thirty-five; thirty-six; thirty-seven; thirty-eight; thirty-nine.

Erase the 1 star and make another row of 10 stars. How many stars are there in 4 rows?

Make 3 stars beneath the 4 rows. How many are 40 stars and 3 stars? 40 stars and 5 stars? 40 stars and 9 stars?

In a column make the figures for forty; forty-one; forty-two; forty-three; forty-four; forty-five; forty-six; forty-seven; forty-eight; forty-nine. What does the word *forty* mean?

Erase the 3 stars and make another row of 10 stars. How many stars are there in 5 rows?

Make 8 stars beneath the 5 rows. How many stars are 50 stars and 8 stars? 50 stars and 5 stars? 50 stars and 2 stars?

In a column make the figures for fifty; fifty-one; fifty-two; fifty-three; fifty-four; fifty-five; fifty-six; fifty-seven; fifty-eight; fifty-nine. What does the word *fifty* mean?

Erase the 8 stars and make a row of 10 stars. How many stars are there in 6 rows?

Make 6 stars beneath the 6 rows. How many stars are there in 6 rows and 6 stars? In 6 rows and 9 stars? What does the word *sixty* mean?

In a column make the figures for sixty, and for all the numbers from sixty-one to sixty-nine.

Erase the 6 stars and make a row of 10 stars. How many rows of 10 stars are there now? How many stars are there in 7 rows? How many are 7 times 10 stars? How many times 10 stars are there in 70 stars? What does the word *seventy* mean?

Make 4 stars beneath the 7 rows of stars? How many are 70 stars and 4 stars? 70 stars and 6 stars? 70 stars and 1 star?

In a column make the figures for all the numbers from seventy to seventy-nine.

Erase the 4 stars and make another row of 10 stars. How many stars are there in 8 rows of 10 stars each? What does the word *eighty* mean? How many times 10 stars are there in 80 stars?

Make 5 stars beneath the 8 rows. How many are 80 stars and 5 stars? How many are 80 stars and 1 star? 80 stars and 2 stars?

In a column write in figures all the numbers from eighty to eighty-nine.

Erase the 5 stars and make another row of 10 stars. How many rows have you made? How many stars? What does the word *ninety* mean? How many are 9 times 10 stars? —— what?

Make 9 beneath the 9 rows. How many stars have you on your slate now? How many are 90 stars and 9 stars? 90 stars and 1 star?

In a column write in figures all the numbers from ninety to ninety-nine. What is the largest number you can write with two figures? Why?

Write in words the names of the numbers for which these figures stand:

23
17
56
37
84
99
27
76
91

62
71
35
48
19
67
53
45
22

96
28
16
55
79
82
15
98
61

66
43
94
18
25
77
34
83
36

Copy and fill blank spaces:

20 stars and 7 stars are —— stars.
40 pens and 4 pens are —— pens.
70 apples and 5 apples are —— apples.
90 books and 9 books are —— books.
30 desks and 6 desks are —— desks.
50 men and 8 men are —— men.
60 trees and 3 trees are —— trees.
80 sheep and 2 sheep are —— sheep.
10 oranges and 8 oranges are —— oranges.
90 pencils and 5 pencils are —— ——.
50 dollars and 1 dollar are —— ——.
70 horses and 9 horses are —— ——.
40 pupils and 6 pupils are —— ——.
60 feet and 7 feet are —— ——.
80 pears and 4 pears are —— ——.
30 cows and 2 cows are —— ——.

LESSON 47.

ORAL EXERCISE.

NOTE.—Save time by having a sufficient supply of bunches of ten splints each, ready for this lesson, before the class is called.

All pass to the board. Make the figures which tell how many splints I hold up each time. (In this way review the recognizing and writing of several numbers between 20 and 98.)

Come to the table. Take 9 bunches and 9 loose splints. How many splints have you? Write figures for ninety-nine. Take one more loose splint. Put it with the 9 splints. How many loose splints have you now? What do you do with ten loose splints? (Bunch them.) How many bunches have you now? How many splints in one bunch? How many splints in 10 bunches? How many are 10 times 10 splints? How many times 10 splints are there in 100 splints? What is one tenth of 100 splints? Show me.

When we had 10 loose splints what did we do with them? Yes, and when we have 10 bunches with 10 splints in each bunch, what might we do with them? (Put a band around them.) Take one of these larger rubber bands and put it around the 10 bunches. How many bunches have you now? (One large bunch.) How many small bunches are there in this large bunch? What do you call the whole number of splints in the large bunch? (1 hundred.)

(Holding up a bunch of ten.) How many bunches have I here? What figure shall I write for this bunch? (1.) James may write it on the board. Can you tell that James's 1 stands for a bunch, a ten, and not for a loose stick? What else is wanted to show that the 1 stands for one ten? Ellen may write the zero at the right of the 1. The zero shows that James's 1 stands in the *second* place. What stands in the *first* place? What does the 1 in the second place stand for? Show me with splints. What would it have stood for in the first place? Show me. The bunch is how many times as much as the one loose splint? Then, 1 in the second place is worth how many times as much as it would be in the first place? Now (holding up the large bunch) we have another *one* here that is how many times as great as the small bunch? (Ten times.) I wonder if anyone can tell me in what place we can put a figure 1 to stand for this one large bunch? (In the *third* place. If the child does not understand this readily, the teacher must continue the work till he does understand it.) John may write 1 on the board to stand for this large bunch, this hundred, this ten times ten. If John's 1 stands for the large bunch, we must think of it as standing in the third place; but how can we show that it does stand in the third place? When 1 stood for ten, how did we show that it was in the second place? Then, how many zeros must we write to show that this one is in the third place, and stands for one hundred? Bessie may write the two zeros at the right of John's 1 to show that it stands in the third place, and means one hundred.

Now, lay down the large bunch, put two small bunches to the right of it, and four loose sticks to the right of the two bunches.

How many are 8+7? 7+8? 15 less 7? 15 less 8? Answer four similar questions for each combination in Table 1.

Learn Table 1 thoroughly. Then you can give *all* the results in Tables 2 and 3 and in the "sight" addition and "sight" subtraction, at once, from memory.

LESSON 45.

ORAL EXERCISE.

NOTE.—Give a short, rapid review on writing the numbers from 11 to 19.

Take 20 splints. Bunch them by tens. What have you?

Emma may make the figure which stands for the bunches. Why should a zero be put at the right of the 2? (To show that the 2 stands for tens, not for ones.)

Put a loose splint with the 2 bunches. What number have you? Lay them on the table in order. Ruth may make on the board the figure which stands for the 2 bunches. Now make the figure which stands for the loose splint. Where are the bunches? Where is the figure that stands for them? Where is the loose splint? Where is the figure that stands for it? Read what is written on the board. Write the word twenty-one.

Put another splint with the loose splint. What have you? Lay them in order. Oliver may make the figures which stand for 2 bunches and 2 loose splints. Where are the bunches? Where is the figure that stands for them? Where are the loose splints? Where is the figure that stands for them? Read what is written on the board.

(Teach 23, 24, 25, 26, 27, 28, 29, in a similar way. Let the children put another splint with the 9 loose ones. They will understand that a band should be put around the new ten. Let it be put on.)

How many bunches have you now? 10 splints and 10 splints and 10 splints are how many splints? 3 times 10 splints are how many splints? How many times 10 splints are there in thirty

splints? How many times can 10 splints be taken from 30 splints? What will be left? What is one third of 30 splints?

George may make on the board the figure that stands for 3 bunches. Now make the figure that shows that the 3 stands for bunches, not for ones. Where are the bunches? Where is the figure that stands for them? Read what you have written.

Quick answers:

9+7=?
20+3=?
7+8=?
11−3=?
9+8=?
14−8=?
15−7=?
20+5=?
19−9=?
5+4=?
8+5=?
13−4=?
20+7=?
18−9=?
20−9=?
20+9=?
16−9=?
6+9=?
17−10=?
5+ 6=?
9+ 2=?
20+ 6=?
7+ 7=?
8+ 8=?
7+6=?
9+3=?
4+8=?
4+7=?
8+9=?
20−1=?

Write figures for all the numbers from 20 to 30.

(Practice on tables and "sight" work.)

NOTE.—Take great care that in writing the compound names of numbers, as twenty-one, twenty-nine, etc., the hyphen is properly used.

FOR STUDY.

Make 10 squares in a row on your slate. Beneath them make another row of 10 squares. How many squares are 10 squares and 10 squares?

Make the figures which tell how many squares you have made. What does the word *twenty* mean?

Make 1 square beneath the 2 rows of 10 squares. How many squares have you now? How many are 20 squares and 1 square? Beneath the figures for 20 squares make the figures which stand for twenty-one squares.

Make another square beneath the 20 squares. How many are 20 squares and 2 squares?

Beneath the figures that stand for twenty-one squares, make the figures that stand for twenty-two squares.

Make another square beneath the 20 squares. How many are 20 squares and 3 squares?

How many are 8+7? 7+8? 15 less 7? 15 less 8? Answer four similar questions for each combination in Table 1.

Learn Table 1 thoroughly. Then you can give *all* the results in Tables 2 and 3 and in the "sight" addition and "sight" subtraction, at once, from memory.

LESSON 45.

ORAL EXERCISE.

NOTE.—Give a short, rapid review on writing the numbers from 11 to 19.

Take 20 splints. Bunch them by tens. What have you?

Emma may make the figure which stands for the bunches. Why should a zero be put at the right of the 2? (To show that the 2 stands for tens, not for ones.)

Put a loose splint with the 2 bunches. What number have you? Lay them on the table in order. Ruth may make on the board the figure which stands for the 2 bunches. Now make the figure which stands for the loose splint. Where are the bunches? Where is the figure that stands for them? Where is the loose splint? Where is the figure that stands for it? Read what is written on the board. Write the word twenty-one.

Put another splint with the loose splint. What have you? Lay them in order. Oliver may make the figures which stand for 2 bunches and 2 loose splints. Where are the bunches? Where is the figure that stands for them? Where are the loose splints? Where is the figure that stands for them? Read what is written on the board.

(Teach 23, 24, 25, 26, 27, 28, 29, in a similar way. Let the children put another splint with the 9 loose ones. They will understand that a band should be put around the new ten. Let it be put on.)

How many bunches have you now? 10 splints and 10 splints and 10 splints are how many splints? 3 times 10 splints are how many splints? How many times 10 splints are there in thirty

splints? How many times can 10 splints be taken from 30 splints? What will be left? What is one third of 30 splints?

George may make on the board the figure that stands for 3 bunches. Now make the figure that shows that the 3 stands for bunches, not for ones. Where are the bunches? Where is the figure that stands for them? Read what you have written.

Quick answers:

9+7=?
20+3=?
7+8=?
11-3=?
9+8=?
14-8=?
15-7=?
20+5=?
19-9=?
5+4=?
8+5=?
13-4=?
20+7=?
18-9=?
20-9=?
20+9=?
16-9=?
6+9=?
17-10=?
5+ 6=?
9+ 2=?
20+ 6=?
7+ 7=?
8+ 8=?
7+6=?
9+3=?
4+8=?
4+7=?
8+9=?
20-1=?

Write figures for all the numbers from 20 to 30.

(Practice on tables and "sight" work.)

NOTE.—Take great care that in writing the compound names of numbers, as twenty-one, twenty-nine, etc., the hyphen is properly used.

FOR STUDY.

Make 10 squares in a row on your slate. Beneath them make another row of 10 squares. How many squares are 10 squares and 10 squares?

Make the figures which tell how many squares you have made. What does the word *twenty* mean?

Make 1 square beneath the 2 rows of 10 squares. How many squares have you now? How many are 20 squares and 1 square? Beneath the figures for 20 squares make the figures which stand for twenty-one squares.

Make another square beneath the 20 squares. How many are 20 squares and 2 squares?

Beneath the figures that stand for twenty-one squares, make the figures that stand for twenty-two squares.

Make another square beneath the 20 squares. How many are 20 squares and 3 squares?

How many are 8+7? 7+8? 15 less 7? 15 less 8? Answer four similar questions for each combination in Table 1.

Learn Table 1 thoroughly. Then you can give *all* the results in Tables 2 and 3 and in the "sight" addition and "sight" subtraction, at once, from memory.

LESSON 45.

ORAL EXERCISE.

NOTE.—Give a short, rapid review on writing the numbers from 11 to 19.

Take 20 splints. Bunch them by tens. What have you?

Emma may make the figure which stands for the bunches. Why should a zero be put at the right of the 2? (To show that the 2 stands for tens, not for ones.)

Put a loose splint with the 2 bunches. What number have you? Lay them on the table in order. Ruth may make on the board the figure which stands for the 2 bunches. Now make the figure which stands for the loose splint. Where are the bunches? Where is the figure that stands for them? Where is the loose splint? Where is the figure that stands for it? Read what is written on the board. Write the word twenty-one.

Put another splint with the loose splint. What have you? Lay them in order. Oliver may make the figures which stand for 2 bunches and 2 loose splints. Where are the bunches? Where is the figure that stands for them? Where are the loose splints? Where is the figure that stands for them? Read what is written on the board.

(Teach 23, 24, 25, 26, 27, 28, 29, in a similar way. Let the children put another splint with the 9 loose ones. They will understand that a band should be put around the new ten. Let it be put on.)

How many bunches have you now? 10 splints and 10 splints and 10 splints are how many splints? 3 times 10 splints are how many splints? How many times 10 splints are there in thirty

splints? How many times can 10 splints be taken from 30 splints? What will be left? What is one third of 30 splints?

George may make on the board the figure that stands for 3 bunches. Now make the figure that shows that the 3 stands for bunches, not for ones. Where are the bunches? Where is the figure that stands for them? Read what you have written.

Quick answers:

$9+7=?$
$20+3=?$
$7+8=?$
$11-3=?$
$9+8=?$
$14-8=?$
$15-7=?$
$20+5=?$
$19-9=?$
$5+4=?$
$8+5=?$
$13-4=?$
$20+7=?$
$18-9=?$
$20-9=?$
$20+9=?$
$16-9=?$
$6+9=?$
$17-10=?$
$5+6=?$
$9+2=?$
$20+6=?$
$7+7=?$
$8+8=?$
$7+6=?$
$9+3=?$
$4+8=?$
$4+7=?$
$8+9=?$
$20-1=?$

Write figures for all the numbers from 20 to 30.

(Practice on tables and "sight" work.)

NOTE.—Take great care that in writing the compound names of numbers, as twenty-one, twenty-nine, etc., the hyphen is properly used.

FOR STUDY.

Make 10 squares in a row on your slate. Beneath them make another row of 10 squares. How many squares are 10 squares and 10 squares?

Make the figures which tell how many squares you have made. What does the word *twenty* mean?

Make 1 square beneath the 2 rows of 10 squares. How many squares have you now? How many are 20 squares and 1 square? Beneath the figures for 20 squares make the figures which stand for twenty-one squares.

Make another square beneath the 20 squares. How many are 20 squares and 2 squares?

Beneath the figures that stand for twenty-one squares, make the figures that stand for twenty-two squares.

Make another square beneath the 20 squares. How many are 20 squares and 3 squares?

How many are 8+7? 7+8? 15 less 7? 15 less 8? Answer four similar questions for each combination in Table 1.

Learn Table 1 thoroughly. Then you can give *all* the results in Tables 2 and 3 and in the "sight" addition and "sight" subtraction, at once, from memory.

LESSON 45.

ORAL EXERCISE.

NOTE.—Give a short, rapid review on writing the numbers from 11 to 19.

Take 20 splints. Bunch them by tens. What have you?

Emma may make the figure which stands for the bunches. Why should a zero be put at the right of the 2? (To show that the 2 stands for tens, not for ones.)

Put a loose splint with the 2 bunches. What number have you? Lay them on the table in order. Ruth may make on the board the figure which stands for the 2 bunches. Now make the figure which stands for the loose splint. Where are the bunches? Where is the figure that stands for them? Where is the loose splint? Where is the figure that stands for it? Read what is written on the board. Write the word twenty-one.

Put another splint with the loose splint. What have you? Lay them in order. Oliver may make the figures which stand for 2 bunches and 2 loose splints. Where are the bunches? Where is the figure that stands for them? Where are the loose splints? Where is the figure that stands for them? Read what is written on the board.

(Teach 23, 24, 25, 26, 27, 28, 29, in a similar way. Let the children put another splint with the 9 loose ones. They will understand that a band should be put around the new ten. Let it be put on.)

How many bunches have you now? 10 splints and 10 splints and 10 splints are how many splints? 3 times 10 splints are how many splints? How many times 10 splints are there in thirty

splints? How many times can 10 splints be taken from 30 splints? What will be left? What is one third of 30 splints?

George may make on the board the figure that stands for 3 bunches. Now make the figure that shows that the 3 stands for bunches, not for ones. Where are the bunches? Where is the figure that stands for them? Read what you have written.

Quick answers:

9+7=?
20+3=?
7+8=?
11-3=?
9+8=?
14-8=?
15-7=?
20+5=?
19-9=?
5+4=?
8+5=?
13-4=?
20+7=?
18-9=?
20-9=?
20+9=?
16-9=?
6+9=?
17-10=?
5+ 6=?
9+ 2=?
20+ 6=?
7+ 7=?
8+ 8=?
7+6=?
9+3=?
4+8=?
4+7=?
8+9=?
20-1=?

Write figures for all the numbers from 20 to 30.

(Practice on tables and "sight" work.)

NOTE.—Take great care that in writing the compound names of numbers, as twenty-one, twenty-nine, etc., the hyphen is properly used.

FOR STUDY.

Make 10 squares in a row on your slate. Beneath them make another row of 10 squares. How many squares are 10 squares and 10 squares?

Make the figures which tell how many squares you have made. What does the word *twenty* mean?

Make 1 square beneath the 2 rows of 10 squares. How many squares have you now? How many are 20 squares and 1 square? Beneath the figures for 20 squares make the figures which stand for twenty-one squares.

Make another square beneath the 20 squares. How many are 20 squares and 2 squares?

Beneath the figures that stand for twenty-one squares, make the figures that stand for twenty-two squares.

Make another square beneath the 20 squares. How many are 20 squares and 3 squares?

How many are 8+7? 7+8? 15 less 7? 15 less 8? Answer four similar questions for each combination in Table 1.

Learn Table 1 thoroughly. Then you can give *all* the results in Tables 2 and 3 and in the "sight" addition and "sight" subtraction, at once, from memory.

LESSON 45.

ORAL EXERCISE.

NOTE.—Give a short, rapid review on writing the numbers from 11 to 19.

Take 20 splints. Bunch them by tens. What have you?

Emma may make the figure which stands for the bunches. Why should a zero be put at the right of the 2? (To show that the 2 stands for tens, not for ones.)

Put a loose splint with the 2 bunches. What number have you? Lay them on the table in order. Ruth may make on the board the figure which stands for the 2 bunches. Now make the figure which stands for the loose splint. Where are the bunches? Where is the figure that stands for them? Where is the loose splint? Where is the figure that stands for it? Read what is written on the board. Write the word twenty-one.

Put another splint with the loose splint. What have you? Lay them in order. Oliver may make the figures which stand for 2 bunches and 2 loose splints. Where are the bunches? Where is the figure that stands for them? Where are the loose splints? Where is the figure that stands for them? Read what is written on the board.

(Teach 23, 24, 25, 26, 27, 28, 29, in a similar way. Let the children put another splint with the 9 loose ones. They will understand that a band should be put around the new ten. Let it be put on.)

How many bunches have you now? 10 splints and 10 splints and 10 splints are how many splints? 3 times 10 splints are how many splints? How many times 10 splints are there in thirty

splints? How many times can 10 splints be taken from 30 splints? What will be left? What is one third of 30 splints?

George may make on the board the figure that stands for 3 bunches. Now make the figure that shows that the 3 stands for bunches, not for ones. Where are the bunches? Where is the figure that stands for them? Read what you have written.

Quick answers:

9+7=?
20+3=?
7+8=?
11−3=?
9+8=?
14−8=?
15−7=?
20+5=?
19−9=?
5+4=?
8+5=?
13−4=?
20+7=?
18−9=?
20−9=?
20+9=?
16−9=?
6+9=?
17−10=?
5+ 6=?
9+ 2=?
20+ 6=?
7+ 7=?
8+ 8=?
7+6=?
9+3=?
4+8=?
4+7=?
8+9=?
20−1=?

Write figures for all the numbers from 20 to 30.

(Practice on tables and "sight" work.)

NOTE.—Take great care that in writing the compound names of numbers, as twenty-one, twenty-nine, etc., the hyphen is properly used.

FOR STUDY.

Make 10 squares in a row on your slate. Beneath them make another row of 10 squares. How many squares are 10 squares and 10 squares?

Make the figures which tell how many squares you have made. What does the word *twenty* mean?

Make 1 square beneath the 2 rows of 10 squares. How many squares have you now? How many are 20 squares and 1 square? Beneath the figures for 20 squares make the figures which stand for twenty-one squares.

Make another square beneath the 20 squares. How many are 20 squares and 2 squares?

Beneath the figures that stand for twenty-one squares, make the figures that stand for twenty-two squares.

Make another square beneath the 20 squares. How many are 20 squares and 3 squares?

At 1 cent a gill, what will a pint of vinegar cost?

At 3 cents a pint, what will a quart of milk cost? 3 quarts?

What will a quart of milk cost at 20 cents a gallon?

What will a pint of vinegar cost at 16 cents a quart?

At 10 cents a quart, what will a gallon of molasses cost?

At 2 cents a pint, what will a gallon of kerosene cost?

How many pints of cream at 10 cents a pint can be bought for 20 cents?

What will 3 quarts of oil cost at 6 cents a quart?

At 8 cents a gallon, what will a quart of skim-milk cost?

If a quart bottle of ink costs 50 cents, how much must I pay for a pint?

LESSON 50.

Dry Measure.

ORAL EXERCISE.

NOTE.—The teacher should have proper measures at hand—a pint, quart, peck, and half bushel. There should also be a sufficient quantity of oats, shelled corn, sand, or something else, to measure.

Who has seen measures like these used? Where have you seen them used, Willie? For what are such measures used, Frank? (For measuring fruits, roots, vegetables, and grains.) What is the smallest measure called? Yes, that is the pint measure. It has the same name as the measure we used in measuring water, but it *is a little larger.* What is the next larger measure?

Nancy may find how many pints of oats it takes to fill the quart measure. How many? Grant may write it on the board in two ways. (2 pints make 1 quart. 1 quart=2 pints.) Is this quart larger than the quart we used before? How do you know? The measure next larger than the quart measure is called a peck measure. Name some articles that are bought and sold by the peck, Roy.

Lewis may find how many quarts of oats it takes to fill the peck measure. Carl may go to the board and keep count. How many quarts does it take to make a peck, Carl? Carl may write, "8 quarts make 1 peck," and "1 peck=8 quarts."

How many quarts in 2 pecks? In one half of a peck? In one fourth of a peck? In one eighth of a peck?

How many quarts in 1 peck and 4 quarts? In 2 pecks? In 2 pecks and 2 quarts?

How many pecks in 8 quarts? In 10 quarts? In 12 quarts? In 16 quarts?

Who knows what the largest measure is called? Name some articles that are bought and sold by the bushel, George. George may find how many pecks of oats it takes to make a half bushel. Now find how many it would take to make a whole bushel? Hattie may write, "4 pecks make a bushel," and "1 bushel=4 pecks."

How many pecks in 2 bushels? In 4 bushels and 3 pecks? In 6 bushels?

How many bushels in 8 pecks? In 6 pecks? In 4 pecks? In 9 pecks?

Mary and Susan went berrying and picked 16 pints of blackberries. They sold them for 10 cents a gallon. How many cents did they receive?

Quick answers:

How many inches in 1 foot? In one half of a foot?
How many feet in 1 yard? In 3 yards? In 4 yards?
How many yards in 3 feet? In 6 feet? In 12 feet?
How many pints in 2 quarts? In 4 quarts? In 6 quarts?
How many quarts in 6 pints? In 10 pints? In 14 pints?
How many quarts in 2 gallons? In 4 gallons and 2 quarts?
How many gallons in 4 quarts? In 6 quarts? In 2 quarts?
How many gallons are there in 8 pints? In 16 pints?

Read the table that has been written on the board.

FOR STUDY.

PINT.

QUART.

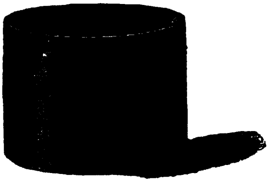

PECK.

TABLE OF DRY MEASURE.

2 pints make 1 quart.
8 quarts make 1 peck.
4 pecks make 1 bushel.

1 quart=2 pints.
1 peck=8 quarts.
1 bushel=4 pecks.

How many pints in 2 quarts? In 4? In 6? In one half of a quart? In 8 quarts? In 9?

How many quarts in 4 pints? In 6 pints? In 7 pints? In 9 pints? In 14 pints? In 20 pints?

How many quarts in 2 pecks? In 2 pecks and 4 quarts? In one half of a peck? In one fourth of a peck? In one eighth of a peck?

How many pecks in 8 quarts? In 10 quarts? In 12 quarts? In 16 quarts?

What part of a peck is 1 quart? 4 quarts?

How many pecks in 2 bushels? In 3 bushels and 3 pecks? In 4 bushels? In 4 bushels and 1 peck? In 5 bushels?

How many bushels in 4 pecks? In 8 pecks? In 10 pecks? In 12 pecks? In 20 pecks?

What part of a bushel is 1 peck? 2 pecks?

A party of boys went chestnuting and found 2 pecks and 2 quarts of nuts. How many quarts did they find?

John feeds his pony 1 peck of oats a day. How many bushels does the pony eat in 12 days?

LESSON 51.

Square Measure.

ORAL EXERCISE.

You may each take a foot-rule and go to the board. Draw a horizontal (explain) line one foot long. From the left-hand end of this line, draw a vertical (explain) line downward, one foot long. From the other end of the horizontal line draw a vertical line downward, one foot long. Connect the lower ends of the vertical lines by a horizontal line one foot long.

How many sides has the figure you have drawn? How do they compare in length? (They are equal in length.) How many corners has the figure? How do they compare in size? (They are equal, or "square" corners.) What is a figure with four equal sides and four equal corners called? (A square.) How long are the sides of the square you have drawn? (One foot.) This is called a square foot. What is a square foot? (It is a square with sides one foot long.)

Draw a horizontal line one inch long. From each end of this line draw a vertical line one inch long. Connect the ends of these vertical lines by another horizontal line one inch long. What kind of a figure have you? What kind of a square is it? (A square inch.) What is a square inch? (It is a square figure whose sides are one inch long.)

How many square inches are there in a figure one inch long and one inch wide?

Make the horizontal lines each one inch longer to the right. Connect the ends by a vertical line one inch long. What kind of a square have you added to the square inch? (Another square inch.) How long is the whole figure? How wide is it? How many square inches are there in a figure two inches long and one inch wide?

Make the horizontal lines each one inch longer to the right. Connect the ends, etc. How long is the whole figure now? How wide is it? How many square inches are there in a figure three inches long and one inch wide?

How many square inches will there be in the figure if we make it four inches long and one inch wide?

Draw a figure four inches long and three inches wide. Mark it off into square inches. How many square inches in one row four inches long and one inch wide? How many such rows?

How many square inches in a figure four inches long and three inches wide? How many times four square inches?

Draw a figure four inches long and four inches wide. We want to find how many square inches there are in this figure without marking it off into square inches. How many strips one inch wide are there, Philip? (Four.) Why do you think so? ("Because each strip is only one inch wide, and the figure is four inches wide.") How many square inches are there in a strip four inches long and one inch wide, Joe? How many square inches are there in four such strips? How many times four square inches?

John may take this yardstick, and draw a square with sides just as long as the stick. What kind of a square have you drawn?

Harry may take his foot-rule and mark off each side of the square yard into feet. Put a dot at the end of each foot.

Lucy may take the yardstick and connect the lower dots on the vertical sides of the square yard by a straight line. Now do the same with the two upper dots.

Nettie may take the yardstick and connect the right-hand dots on the horizontal lines by a straight line. Do the same with the left-hand dots. Roy may measure the sides of one of the smaller squares. What kind of a square is it? (A square foot.) Are the small squares the same size? What kind of squares are they? (Square feet.) How many small squares are there in the large one? How many square feet make a square yard?

Bessie may write, "9 square feet make 1 square yard," and "1 square yard=9 square feet."

FOR STUDY.

When we measure surfaces we use square measure.

Make a square inch neatly on your slate. Use the inch on your foot-rule to measure with. Make another square 2 inches on a side. See if you can draw some lines across the larger square so as to show how many square inches it contains.

TABLE FOR SQUARE MEASURE.

144 sq. inches make 1 sq. foot.	1 sq. foot=144 sq. inches.
9 sq. feet make 1 sq. yard.	1 sq. yard=9 sq. feet.

How many square feet in 2 square yards? In 2 square yards and 2 square feet? In 1 square yard and 4 square feet? How many square feet in one third of a square yard?

How many square yards are there in 18 square feet? In 12 square feet?

Draw on your slate a figure with square corners, 3 inches long and 1 inch wide. Mark it off into square inches. How many square inches in a figure 3 inches long and 1 inch wide?

How many square inches would there be in the figure if it were 5 inches long and 1 inch wide? If it were 9 inches long and 1 inch wide? If it were 12 inches long and 1 inch wide?

How many square feet in such a figure 1 foot long and 1 foot wide? In such a figure 2 feet long and 1 foot wide? 4 feet long and 1 foot wide? 8 feet long and 1 foot wide?

Draw a figure with square corners, 3 inches long and 2 inches wide. Mark it off into square inches. How many square inches in a strip 3 inches long and 1 inch wide? How many in two such strips? How many in such a figure 6 inches long and 2 inches wide? How many times 6 square inches? In such a figure 7 inches long and 2 inches wide? In such a figure 8 inches long and 2 inches wide? In such a figure 10 inches long and 2 inches wide? How many times 10 square inches?

How many square feet in such a figure 3 feet long and 2 feet wide? In one 4 feet long and 2 feet wide? On one side of a board 6 feet long and 2 feet wide? How many times 6 square feet?

Draw such a figure 4 inches long and 3 inches wide. Mark it off into square inches. How many square inches are there in the figure 4 inches long and 3 inches wide? How many times 4 square inches? How many times 3 square inches? Show it by the figure.

How many square inches would there be if it were 4 inches long and 2 inches wide? If it were 4 inches long and 4 inches wide? If it were 5 inches long and 4 inches wide? How many times 5 square inches? How many times 4 square inches? Show it by the figure.

How many square feet in a piece of oilcloth 5 feet long and 4 feet wide?

How many square inches on one side of an envelope 5 inches long and 3 inches wide?

How many square yards in a floor 5 yards long and 4 yards wide?

How many square yards would there be in the floor if it were 6 yards long and 6 yards wide?

How many square inches in one side of a crayon box 6 inches long and 3 inches wide?

The top of my desk is 4 feet long and 3 feet wide. How many square feet of leather will be required to cover it?

How many square feet in a piece of zinc 4 feet long and 4 feet wide? How many times 4 square feet?

LESSON 52.

Measures of Volume.

ORAL EXERCISE.

NOTE.—For apparatus, the teacher should have several one-inch cubes, and a one-foot cube. A few other rectangular solids would be serviceable. These may be obtained ready prepared at a small expense; or some of the most careful boys will be glad to saw them out under the teacher's direction.

Each of you may take one of the small blocks. How many faces (sides) has it? How many edges? How many corners? Show me. Lay it with one face toward you. In how many directions can it be measured? (Three.) What is one direction, Albert? (From left to right.) What is another, Laura? (From top to bottom.) Another, Bessie? (From front to back.) Is there any other direction in which it can be measured? Take your rules and tell me how long it is from left to right. How high is it? How wide from front to back?

What kind of a face is the one next to you? (A square.) What kind of a face is the one on top? All the other faces? How many square faces has this block? How do they compare in size? (They are equal.) This solid is called a cube. Are there any more cubes among the solids on the table? Clara may show me one. John may show me another. What is a cube? (A solid whose faces are all squares, and whose corners are all equal.) How many faces has a solid?

Measure the top face of this cube. What kind of a square is it? (A square inch.) Are the other faces just like it? What kind of faces are they? (They are one-inch squares.) A cube whose faces are all one-inch squares is called a cubic inch.

Elmer may take the large cube. Measure one face. What kind of a square is it? (A square foot.) What shall we call a cube whose six faces are one foot square? (A cubic foot.) What is a cube whose faces are one yard square? (A cubic yard.)

Robert may take three inch cubes and lay them in a row with faces together. How wide is the row? How high is it? How long is it? If the corners are square, how many cubic inches in a solid one inch thick, one inch wide, and three inches long? Make the row one inch longer. How long is it now? How many cubic inches are there in it? How many cubic inches would there be in it if it were eight inches long? How many cubic inches in such a solid one inch thick, one inch wide, and ten inches long?

Henry may make a solid one inch thick, one inch wide, and three inches long with the inch cubes. How many cubic inches are there in it? How many cubic inches would there be in it if it were two inches wide? (Two times three cubic inches.) Henry may show me that it would be so. How many cubic inches would there be in it if it were four inches wide? (Four times three cubic inches.) If it were five inches wide? Show me.

We have a solid here one inch thick, three inches wide, and three inches long. How many cubic inches are there in it? (Nine cubic inches.) How many cubic inches would there be in it if it were two inches thick? (Two times nine cubic inches.) Ida may show me that it would be so. How many cubic inches would there be in it if it were three inches thick? (Three times nine cubic inches.) How many is that? What kind?

I want to know how many cubic inches there are in this solid (showing a solid two inches by two inches by four inches). George may find how thick, how wide, and how long it is. How many cubic inches are there in one row one inch thick, one inch wide, and four inches long? How many cubic inches are there in one layer one inch thick, four inches long, and two inches wide? (Two times four cubic inches.) How many cubic inches are there in two layers one inch thick, two inches wide, and four inches long? (Two times eight cubic inches.) How many is that? What?

FOR STUDY.

How many edges has a cube? How many corners? How many faces? What kind of faces? What kind of corners?

What kind of faces has a 1-inch cube? What kind of faces has a 1-foot cube? What kind of faces has a 1-yard cube?

How many square inches of paper would be needed to cover a 1-inch cube? How many would be needed to cover a 1-foot cube? Why?

How many cubic inches are there in a solid 1 inch thick, 1 inch wide, and 1 inch long, if the corners are all square? 1 inch thick, 1 inch wide, and 2 inches long? 4 inches long? 5 inches long? 7 inches long?

If the corners are all square, how many cubic feet are there in a solid 1 foot thick, 1 foot wide, and 1 foot long? How many in a solid 2 feet long? 3 feet long? 6 feet long? 10 feet long? How do you know?

How many cubic feet in a block of wood 1 foot thick, 1 foot wide, and 5 feet long, having square corners? In a block of wood 1 foot thick, 2 feet wide, and 5 feet long? 1 foot thick, 3 feet wide, and 5 feet long? 1 foot thick, 4 feet wide, and 5 feet long? How do you know?

How many cubic inches in a block of marble 1 inch thick, 3 inches wide, and 3 inches long, if the corners are square? In a block that is 2 inches thick, 3 inches wide, and 3 inches long? How do you know?

How many cubic feet are there in a block of granite having square corners, 2 feet thick, 2 feet wide, and 2 feet long? In two such blocks? In three such blocks?

How many square feet of paper would be needed to cover 1 face of this block? How many square feet would be needed to cover 3 faces? To cover 6 faces? Show by drawing.

How many cubic inches in a brick 8 inches long, 4 inches wide, and 1 inch thick? How many cubic inches in two such bricks?

LESSON 53.

Avoirdupois Weight.

ORAL EXERCISE.

NOTE.—A pair of scales and some sand are needed for this lesson. If the school is not provided with scales, the teacher can borrow a pair.

Who has seen scales like these somewhere else? Where have you seen them, Fred? Who has seen them used? For what have you seen them used, Mary? Who knows how to use them? What figure do you see on the smallest weight? What is this weight called? (An ounce weight.) What figure do you see on the largest weight? What is this weight called? (A pound weight.)

(Show the 2-ounce, 4-ounce, and 8-ounce weights, and tell the children what they are.)

Agnes may weigh out one ounce of sand. Now two ounces. Roy may weigh out three ounces, etc.

Now that we know how to use the weights, we want to know how many ounces it takes to make a pound. Do you think nine ounces are more or less than one pound? Hugh may weigh one ounce of sand and put it into this bag. Now he may keep on weighing out one ounce and putting it into the bag until there are ounces enough in the bag to balance the pound weight, or weigh one pound. Well, Hugh, what have you learned? (Hugh: "I have learned that it takes sixteen ounces to weigh one pound.") Ralph may write on the board, "16 ounces make 1 pound," and "1 pound=16 ounces."

Lena may weigh out a pound and two ounces; a pound and three ounces; a pound and four ounces; a pound and five ounces.

In weighing twelve ounces, which weights must you use, Mary? In weighing sixteen ounces, which must you use?

Margaret's mother sent her to the store to buy one fourth of a pound of tea. How many ounces did she buy? How many ounces in one fourth of a pound?

Who knows how coal and hay are bought and sold? (By the ton.) Two thousand pounds make one ton. George may write it on the board, in the right place: "2,000 pounds make 1 ton. 1 *ton=2,000 pounds.*"

FOR STUDY.

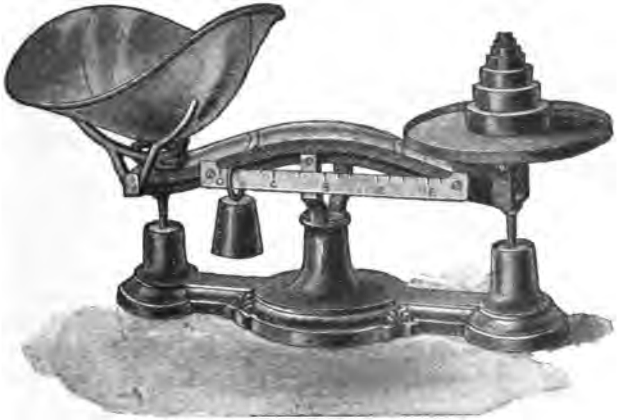


TABLE.

16 ounces make 1 pound.
2000 pounds make 1 ton.

1 pound=16 ounces.
1 ton=2000 pounds.

How many ounces are there in one half of a pound? One fourth? In one eighth?

What will 5 pounds of sugar cost at 4 cents a pound? How many times 4 cents?

What will one fourth of a pound of spice cost at 2 cents an ounce? Why?

I paid 18 cents for 3 pounds of soap. What did 1 pound cost? How do you know?

What will 8 ounces of butter cost at 20 cents a pound? Why?

5 pounds of sweet potatoes cost 20 cents. What did 1 pound cost? 2 pounds and 8 ounces? How do you find it?

What will one fourth of a pound of mutton cost at 12 cents a pound?

LESSON 54.

Measures of Time.

ORAL EXERCISE.

NOTE.—It is desirable to have a dummy clock-dial, with movable hands, and some calendars also.

How many hands has a clock? What are they called? Yes, they are sometimes called "long hand" and "short hand," and sometimes "minute hand" and "hour hand." Which is the minute hand? Why is the short hand called the hour hand? Which moves the faster? How do the hands tell us when it is one o'clock? Show on this clock face. When it is two o'clock? Show. Four o'clock? Seven o'clock? Twelve o'clock? Show me. How long does it take the hour hand to go from I to II? From V to VI? How far does the minute hand go while the hour hand moves over one of these spaces? (Goes once around the face of the clock.) How long, then, does it take the minute hand to move once around the face of the clock? (One hour.) Who knows how many minutes it takes the minute hand to move from XII to I? (Five minutes.) How many minutes does it take the minute hand to move from II to III? How many minutes does it take the minute hand to move from any group of letters to the next?

Into how many small parts is the space between XII and I divided? (Five.) The space between I and II? How many of these small spaces are there around the whole face of the clock? Count them. (Sixty.) How long does it take the minute hand to pass over them all? (One hour.) Yes, and it passes over one of them in a minute. Then how many minutes are there in one hour? Robert may write on the board, "60 minutes make one hour," and "1 hour=60 minutes."

When the minute hand is at I, how many minutes past the hour is it? When it is at II? At III? At IV?

Who can count by fives? Ray may do it. That is an easy way to count, and you must all learn to count that way before you can tell the time by a clock or a watch. All count by fives to sixty.

NOTES.—Give the children a short drill in counting by fives to sixty, by pointing to each group of letters in turn, beginning with I.

If you have the dummy dial, give problems by setting the hands in different positions for the pupils to read them. What is the time when the minute hand is at V and the hour hand a little past II? Give examples of time, and let the pupils set the hands to express the time. Set the hands for nine o'clock and thirty-five minutes, etc.

Who knows what this little hand on my watch is called? (Second hand.) How many minutes does it take this hand to move once around? (One minute.) How many seconds does it take this hand to move once around? Count on this little dial. (Sixty seconds.) How many seconds are there in a minute, then? Alice may write above what is on the board, "60 seconds make 1 minute," and "1 minute=60 seconds."

If it takes the hour hand one hour to move from one group of letters to the next, how many hours does it take it to move once around the face of the clock? (Twelve.) At what time in the night does the hour hand point to XII? (At midnight.) When does it point to XII next? (At noon, next day.) How many hours are there between midnight and noon? (Twelve.) When does it point to twelve again? (Midnight, next night.) How many times does the hour hand move around the face of the clock from midnight to midnight? How many hours does it take each time? How many hours are twelve hours and twelve hours? How many hours, then, from midnight to the next midnight? What do we usually call the time from midnight to the next midnight? (One day.) How many hours are there in a day? Martha may write beneath what is on the board, "24 hours make 1 day," and "1 day=24 hours."

Tell me the names of all the days of the week in their order. How many days did she name? How many days in a week? Julia may write, "7 days make 1 week," and "1 week=7 days."

Name in order the months in a year. How many are there? How many days are there in a month? Here are some calendars. What is the first month of the year? Find it while I write the name on the board. How many days has it? I will write 31 after the name of the month. What is the next month? How many days has it?

(Go through the list, writing the names and the number of days for each month, or let the pupils write them.)

How many *months* have thirty-one days? Which are they?

How many have thirty days? Which are they? Which one has some other number? How many days has February? Does it always have twenty-eight days? It has twenty-eight days for three years, and the fourth year it generally has twenty-nine days. This is called Leap Year. For business transactions, thirty days are counted for a month. But you ought to learn and remember the number of days in each month. This little couplet will help you to remember the months that have thirty days:

"Thirty days has September,
April, June, and November."

How many days has February? How many days each have the other seven months? How many months are there in a year? Bert may write the figures on the board.

(Take a little time for practice on tables, "sight" work, and rapid oral combinations at each lesson.)

FOR STUDY.

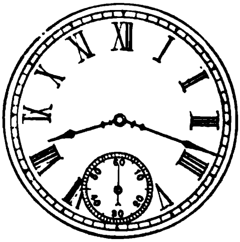


TABLE.

60 seconds make 1 minute.	1 minute=60 seconds.
60 minutes make 1 hour.	1 hour=60 minutes.
24 hours make 1 day.	1 day=24 hours.
7 days make 1 week.	1 week=7 days.
12 months make 1 year.	1 year=12 months.

A man walked 4 miles the first hour, 3 miles the second, and 2 miles the third. How many miles did he walk in 3 hours?

How many days are there in 2 weeks?

How many weeks will it take me to save 18 dollars if I save 2 dollars a week?

How many weeks are there in 14 days?

If a train goes a mile a minute, in how many hours will it go 60 miles?

How many days are there in 2 weeks and 5 days?

There are 5 school-days in a week. If a boy walks 3 miles a day in attending school, how many miles does he walk in a week? How many times 3 miles?

A boy who picks 4 quarts of berries in an hour, has picked 12 quarts. How many hours was he picking?

If it takes a boy 1 minute to slide down a hill and 4 minutes to walk up, how many times can he slide down and return in an hour?

Count by fives to 60.

Draw the face of a clock on your slate. To what group of letters must the minute hand point at 1 o'clock and 20 minutes? At 3 o'clock and 35 minutes? At 3 o'clock and 50 minutes? Show on your drawing.

How many minutes past the hour is it when the minute hand points to IIII? When it points to VI? When it points to IX?

Write the names of the days of the week.

Write the names of the months of the year in order, and after each write the number of days in that month.

What time is it when the hour hand points between IIII and V and the minute hand points to X? (Say 4 o'clock and 50 minutes.)

Draw a clock face and show how the hands will stand at 6 o'clock and 25 minutes.

LESSON 55.

Fractions.

ORAL EXERCISE.

NOTE.—Do not begin the study of fractions by the dividing of a unit, nor emphasize *dividing* at any time. Let the thought of making wholes be the dominant thought, instead. No doubt the word fraction comes from a Latin word, which means to break, but fractions are not "broken units." They are *relative* units, a certain number of which will make a *primary* unit. When written, the numerator shows how many relative units are given, and answers the question, how many? The denominator shows how many of these relative units will make a primary unit, and so names them, or answers the question, what kind. The denominator does for the common fraction what the position of the figures does in our decimal system, in telling what kind of units each figure stands for, as hundredths, tenths, etc. The pupil who has mastered the previous lessons of this book is already familiar with the fractional *idea*, but he has not learned the notation, nor has he learned to perform the special operations which belong to fractions. Nine twelfths does not differ from any other nine except in its limitations—in its relation to a higher unit. The pupil should learn first, and always, that the *numerator expresses the number;* the denominator merely limits it, or tells what kind it is. There is no better apparatus for teaching the first steps in fractions than coins. They always interest the pupils, and they are already more or less familiar with them.

(Holding up three quarter-dollars.) How many do I hold up? Tom, make a figure on the board for the number I hold up. What are these? (Probably some one will say, "Quarters.") Can you think of any other word that means the same as quarter? (Fourth.) Why do you call these fourths, or quarters? (Because four of them are worth one dollar.) Tom, you may draw a short horizontal line under your 3, and write under it the figure that shows how many ones of this kind are worth a dollar. Mary, point to the figure that shows how many I have. Point to the figure that shows how many of these are worth one dollar. How many do I hold up now? (Showing two quarters.) Write a figure for the number. How many are worth one dollar? Write the figure for that in the proper place. Which did I change, the number or the kind? Which figure is changed in the writing? Which is not changed? Can you tell me why this is so? How many do I show you now? (Showing four dimes.) Write the figure for this number. How many of this kind will make a dollar? Write, in the proper place, figures which will tell how many of this kind make a dollar. If ten of this kind are worth one dollar, what should I call them? Yes, they are tenths of a dollar, although we commonly call them *dimes.*

Peter will write on the board the figure for five. Now write another figure that tells that five of the kind 5 stands for would make a whole one of a larger kind. See what I write (writing $\frac{4}{9}$.) How many does this writing stand for? How many of this kind would make a whole one of a larger kind? How do you know that nine of this kind will make a whole one of a larger kind? If nine like these will make that larger one, what kind is this four? (Ninths.) How many loose sticks made a bunch? Write 7 to stand for loose sticks. Now write with it the figures that tell how many make a bunch. What part of a bunch is seven sticks? (Seven tenths of a bunch.) Why? (Because ten of them will make one whole bunch.)

Now read these expressions and tell what each means (writing $\frac{3}{4}$, $\frac{4}{10}$, $\frac{2}{7}$, $\frac{5}{8}$, etc.) (Model: $\frac{3}{4}$ means three ones of such a kind that four of them would make a larger one, and so they are called fourths, etc.) Expressions such as I have written on the board are called *fractions.* Where, in a fraction, is the figure which tells how many ones we have? (Above the line.) Yes, and because this figure tells the number, it is called the *numerator,* which means *numberer.* Where is the figure which tells how many like these ones will make a larger one? If it takes seven to make a larger one, what do you call them? If it takes nine? Five? Twenty? etc. Which figure, then, tells what kind the number is — what name it has? Yes, and so the figure below the line is called the *denominator,* which means *namer.*

FOR STUDY.

Write a 3 to stand for pecks. Draw a short horizontal line beneath it. Under the line write the figure which tells how many pecks there are in a bushel. How many does this writing stand for? Which figure tells it? How many of this kind will make a larger one of another kind?

Which figure tells this? What part of a bushel is 1 peck? 3 pecks? Is that as much as a whole bushel? How do you know?

Write a 9 to stand for inches. In the proper place, write the figures which tell how many inches will make a foot. How many does this fraction stand for? Which figure tells it? How many of this kind make a larger one of another kind? Which figures tell this? If 12 of this kind will make a larger one, what kind is the 9? (Twelfths.) What part of a foot are 9 inches?

Write a 12 to stand for ounces. In the proper place write the figures which tell how many ounces make a pound. How many does this fraction stand for? Which figure tells it? How many of this kind make a larger one of another kind? Which figures tell this? If 16 of this kind make a larger one, what ought the 12 to be called? (Sixteenths.) What part of a pound are 12 ounces?

Read the following fractions. In each case tell:

First. How many ones are in the number.

Second. How many of the kind given make a larger one.

Third. What kind they are.

$\frac{3}{5}$ $\frac{7}{8}$ $\frac{1}{6}$ $\frac{6}{8}$ $\frac{8}{9}$ $\frac{11}{18}$ $\frac{4}{5}$ $\frac{3}{9}$ $\frac{8}{10}$ $\frac{9}{15}$ $\frac{10}{17}$ $\frac{9}{12}$ $\frac{7}{15}$ $\frac{4}{11}$ $\frac{1}{14}$

(Model for the first: "There are three ones in the number; five of this kind will make a larger one, and so they are called fifths.")

Write the following in figures:

Eight ninths.
Five elevenths.
Seven eighths.
Six twelfths.
Ten thirteenths.
Four fifteenths.
Nine nineteenths.
Seven sixteenths.
Eleven twelfths.
Fourteen seventeenths.
Eight fourteenths.
Twelve eighteenths.
Two twentieths.
Thirteen sixteenths.

Which is the numerator in each of the fractions you have just written? Why? Which is the denominator? Why?

Write 2 thirds. Now write 2 fourths. Which has the larger denominator? Which is worth the more? Then, if the numerators are the same, is the fraction with the larger or smaller denominator worth the more? Why?

If your mother is going to give you 1 piece of mince pie, would you rather she should cut the pie into quarters or sixths? Why?

LESSON 56.

Fractions—Continued.

ORAL EXERCISE.

NOTE.—The wooden cubes subdivided into halves, quarters, etc., which are for sale by all dealers in material for primary schools, are excellent objects to use in teaching the elementary truths of fractions. In case the teacher can not procure these, neatly made circles of paper, divided and subdivided, are good. Common objects, like apples, may be divided and the parts used in the same way; but a serious objection to the use of such articles is that the *ones are not all of the same size.* Carefully drawn lines on the blackboard, of the same length, and having divisions marked upon them for halves, thirds, fourths, sixths, etc., will serve an excellent purpose when skillfully used. The object of the following lesson is to show that multiplying or dividing both terms of a fraction by the same number does not change its value. Lines are used for illustration. Have two lines of equal length on the board; have one of them correctly marked off into fourths, the other into thirds.

What number do I show you? (Showing two thirds of a circle.) Peter, write the figure for this number on the board. What are these parts of? Are these parts the same size? Mary, examine them and find out. How many parts like these will make a whole circle? Mary may write a 3 near Peter's 2, so as to show that three parts like those the 2 stands for will make a larger whole one. Ellen may put three of these parts together, and show that they make a whole circle. Look at this line. How many parts in it? Are they equal? Measure them and see. How many of them do I check? (Putting little crosses on two.) Silas may write a figure to show how many I have checked. How many of these equal parts make a whole line? Silas, write the figure by your 2 that will show that four parts like the two will make a whole one. Class, read what Silas has written. Jane, point to the numerator of the fraction. Class, read it. Jane may point to the two parts the 2 stands for. What is the denominator? What does it show? Let us tie these two parts together, so. How many have we now instead of two? (One.) How many parts like this one would be equal to a whole line? Just at the right of Silas's $\frac{2}{4}$, Tom may write the figures to show what we now have. ($\frac{1}{2}$.) Point to the part of the line that the $\frac{2}{4}$ stood for. Henry, point to the part of the line that Tom's ^{1} stands for. Which means the most, then, Silas's $\frac{2}{4}$ or Tom's $\frac{1}{2}$? (They mean just the same.) Then Tom may put the two little lines between them to show that what one means is equal to what the other means. All read it now. (Two fourths equal one half.) Let us look at these numerators and denominators. The last numerator is just half of the first numerator; and the last denominator is just what part of the first denominator? (One half.) And the two fractions mean just ——? (The same thing.) Then, if we make each term of the fraction one half of what it was before, the fraction will mean ——? (The same thing.) Let us try one more fraction without any lines and see how it works. See this. (Writing $\frac{6}{8}$.) How many does this stand for? What kind? John, write a fraction just at the right of this, with a numerator and denominator each one half of those we now have. All read it. Does $\frac{3}{4}$ mean just as much as $\frac{6}{8}$? Who can show me in some way that this is true, $\frac{6}{8}=\frac{3}{4}$? (Have it shown with objects.) See this other line. How many parts has it? Are the parts of equal length? Susan, write the fraction for what I check. (Checking one of the three parts.) What is the numerator? Point to what it stands for. What is the denominator? What does the denominator 3 show? I will cut the one part I checked right in two in the middle. See? How many parts have we now? How many parts like these two will make a whole? Who can show me? (Some one will show by dividing each of the two remaining parts into halves.) How many parts have we? (Pointing to the two sixths.) Edith, write a figure for them. How many such parts will make a whole one? Write the 6 in the right place. Then two sixths equal ——? (One third.) Say so with figures. The new numerator is how many times the old one? The new denominator is how many times the old one? The new fraction is worth just ——? (The same as the old one.) Then, if we make both the numerator and denominator just the same number of times what they were before, the new fraction will be worth ——? (Just the same as the old one.)

FOR STUDY.

Draw a horizontal line two inches long on your slate. Take your foot-rule, divide the line into half inches, and put a dot at the end of each half inch. From each end of this line draw a line downward one and one half inches long, and put a dot at the end of each half inch. Be sure to make the corners square. Connect the lower ends of the vertical lines by a horizontal line two inches long, and put a dot at the end of each half inch. You have drawn a figure which has equal corners, but it is longer one way than it is the other. Such a figure is not a square. Why? It is called an *oblong, or parallelogram*. Spell this hard word.

Connect the lower dots on the vertical lines by a straight line. Connect the upper dots in the same way. Connect the left-hand dots on the horizontal lines by a straight line of dots. Connect the rest of the dots in the same way by lines of dots. Into how many parts have you divided the oblong? Are the parts equal? Measure them and see. How many little squares in the oblong?

Write the figure which tells how many of these squares are in the two top rows. By the 8 write the figures which will show that twelve squares like the eight squares make the whole oblong.

Read what you have written. Which is the numerator? Point to what it stands for. What does the denominator show?

Erase the lines of dots carefully. Instead of eight parts in the two upper rows, how many parts are there now? Just at the right of the 8 write 2. How many parts like these two make the oblong? Write the figure which tells it, in the proper place by the 2. Which means more, $\frac{8}{12}$ of the oblong or $\frac{2}{3}$ of the oblong? Since they mean the same thing, put between them the two little lines which show that the fractions are equal. Read what you have now.

The last numerator is what part of the first numerator? The last denominator is what part of the first denominator? When each term of the fraction is made one fourth of what it was at first, is the worth of the fraction changed? Show on the oblong that $\frac{8}{12}$ is just the same as $\frac{2}{3}$.

Draw another oblong two and one half inches long and two inches wide. Take pains to make the corners square. Mark off each side into half inches by dots. Connect these dots as you did those in the first oblong.

Into how many equal parts is the oblong divided? Write the figures which tell how many of these parts are in the three upper rows. By the 15, in the proper place, write the figures that show that twenty parts like the fifteen parts make the whole oblong.

Read what you have written. Which is the denominator? What does it show? What does the numerator stand for?

Erase the lines of dots carefully. Instead of fifteen parts in the three upper rows, how many parts are there now? Just at the right of the 15 write 3. How many parts like these three parts make the oblong? Write the figure which tells it, in the proper place by the 3. Which means more, $\frac{15}{20}$ of the oblong or $\frac{3}{4}$ of the oblong? Since they mean the same thing, write between them the little lines that show that they are equal. Read what you have now.

The last numerator is what part of the first numerator? The last denominator is what part of the first denominator? By making each term of the fraction one fifth of what it was at first, have you changed the worth of the fraction?

Copy the following fractions by columns, and write after each the sign that means "are" or "equal." After these little signs, write fractions in this way: In the first column make each term of the fractions one half of what it is now; in the second column make each term one third of what it is now; in the third column make each term one fourth of what it is now.

(1)	(2)	(3)
$\frac{6}{8}$	$\frac{3}{9}$	$\frac{4}{12}$
$\frac{10}{14}$	$\frac{6}{12}$	$\frac{8}{16}$
$\frac{12}{16}$	$\frac{12}{15}$	$\frac{4}{20}$
$\frac{18}{20}$	$\frac{15}{18}$	$\frac{16}{20}$

Now read what you have written.

Is $\frac{6}{8}$ equal to $\frac{3}{4}$? How do you know?

Are all of those fractions you have said are equal, really equal? Could you show in some way that they are equal?

Draw another oblong two and one half inches long and two inches wide, and mark off each side into half inches. Connect each pair of dots on the long, horizontal sides only.

Into how many equal parts is the oblong divided? What part of the oblong are the three equal parts at the left-hand side? Write the fraction which will show it. Which figure is the numerator of the fraction? What does it show? Point to these three parts. Which is the denominator? What does it show? Now connect each pair of dots on the sides of the oblong. Into how many equal parts is the oblong divided now? How many of these parts are there in the three left-hand rows? Just at the right of the fraction $\frac{3}{5}$, write the fraction which will show what part of the whole oblong the squares in the three left-hand rows are. Which means more, $\frac{3}{5}$ of the oblong or $\frac{12}{20}$ of it? Since they mean the same, write the proper sign between the fractions. Read what you have written. How do you know that $\frac{3}{5}$ equal $\frac{12}{20}$?

The last numerator is how many times the first numerator? The last denominator is how many times the first denominator? Does it change the worth of a fraction to make each term four times as great as it was at first?

Copy the following fractions by columns, and after each make the sign which means "equal." After these signs in the first column write fractions with terms two times as great as those in the fractions now written. In the second column write fractions with terms three times as great. In the third column write fractions with terms four times as great.

(1)	(2)	(3)
$\frac{6}{7}$	$\frac{2}{5}$	$\frac{1}{3}$
$\frac{5}{8}$	$\frac{3}{4}$	$\frac{2}{4}$
$\frac{4}{9}$	$\frac{2}{3}$	$\frac{3}{5}$
$\frac{6}{10}$	$\frac{5}{6}$	$\frac{1}{2}$

Read what you have written.

Is $\frac{6}{7}$ equal to $\frac{12}{14}$? Are all the other fractions that you have said are equal, really equal? Could you show it?

LESSON 57.

Fractions — Continued.

ORAL EXERCISE.

How many do I show you? (Showing six half-dollars.) What are they? Grace, write the fraction for them on the board. How many like these will make a whole dollar? Which figure shows that two of them will make a dollar? What do you call the 2? Which figure tells how many pieces I have? What do you call the 6?

John, come and take enough of these pieces to make a dollar. How many did he take, Ruth? Sarah, take pieces enough to make another dollar. How many are left? What will they make? What did the denominator tell? How many times could we take two half-dollars out of the six half-dollars? How many times two in six? How many dollars have we? Which is worth more, six half-dollars or three dollars? (They are worth the same.) How do you know? If three dollars are worth the same as six half-dollars, we will say so in figures in this way. (Writing $\frac{6}{2}$ dollars = 3 dollars.) How many whole ones in six halves? How many whole ones in eight halves? Say that in figures. How many twos in eight? How many whole ones in ten halves? Say that in figures. How many twos in ten?

How many do I show you now? (Showing eight thirds of a circle.) What are they? Henry, write the fraction for this number. How many parts like these will make a whole circle, Henry? How many whole circles can you make with these eight thirds? (If any child can tell without counting out, let him do so. If not, count them out as before.) How many pieces are left? How many times three in eight? How many left over? Which is more, eight thirds of a circle or two circles and two thirds? Say, in figures, then, that $\frac{8}{3}$ is equal to $2\frac{2}{3}$. Say it in this way: $\frac{8}{3}=2\frac{2}{3}$. See if you can tell me what $\frac{9}{3}$ equal. $\frac{10}{3}$. I will write $\frac{9}{3}$ and $\frac{10}{3}$ on the board. Charles may write figures to show what they are equal to. How many times three in nine? How many times three in ten?

Complete these expressions:

$\frac{2}{3}=$ $\frac{7}{3}=$ $\frac{8}{4}=$ $\frac{9}{4}=$ $\frac{7}{5}=$ $\frac{10}{3}=$ · etc.

FOR STUDY.

If a pail contains $\frac{15}{2}$ quarts of water, how many quarts does it contain?

Ella gave 8 quarter-dollars in payment for a pair of shoes. How many dollars did the shoes cost?

For his birthday party, Willie had 16 bags of nuts, each holding $\frac{1}{8}$ of a pound. How many pounds had he?

A farmer put $\frac{11}{4}$ of a bushel of apples into a barrel. How many bushels did the barrel hold?

Albert earns $\frac{15}{5}$ of a dollar a week. How many dollars does he earn?

If a man's steps average $\frac{18}{6}$ of a foot, how many feet does he move forward at each step? At 2 steps? At 5 steps?

Hattie bought $\frac{9}{3}$ of a yard of ribbon. How many yards did she buy? How many feet?

Alice made 18 glasses of jelly, each holding $\frac{1}{2}$ pint. How many quarts did she make?

A merchant bought 16 cans of maple syrup, each containing $\frac{1}{4}$ of a gallon. How many gallons did he buy? How many quarts?

Frank earned $\frac{1}{3}$ of a dollar a day for 15 days. How many dollars did he earn? How many times 3 in 15?

A merchant bought 20 cans of baking powder, each containing $\frac{1}{4}$ of a pound. How many pounds of baking powder did he buy?

How many apples in 4 half-apples? In 5 half-apples? In 6 half-apples? In 9 half-apples?

How many pears in $\frac{3}{3}$ of a pear? In $\frac{5}{3}$ of a pear? In $\frac{7}{3}$ of a pear? In $\frac{12}{3}$ of a pear?

How many dollars in $\frac{4}{4}$ of a dollar? In $\frac{5}{4}$ of a dollar? In $\frac{7}{4}$ of a dollar? In $\frac{8}{4}$ of a dollar?

How many bushels in $\frac{7}{7}$ of a bushel? In $\frac{10}{7}$ of a bushel?

How many yards in $\frac{10}{9}$ of a yard? In $\frac{13}{9}$ of a yard?

Copy and complete as in the first two:

$\frac{8}{2}=4$
$\frac{11}{2}=5\frac{1}{2}$
$\frac{12}{2}=$
$\frac{14}{2}=$
$\frac{15}{2}=$

$\frac{9}{3}=$
$\frac{11}{3}=$
$\frac{14}{3}=$
$\frac{15}{3}=$
$\frac{18}{3}=$

$\frac{16}{4}=$
$\frac{18}{4}=$
$\frac{20}{4}=$
$\frac{5}{5}=$
$\frac{8}{5}=$

$\frac{19}{5}=$
$\frac{20}{5}=$
$\frac{7}{6}=$
$\frac{10}{6}=$
$\frac{12}{6}=$

Read what you have written on your slate.

LESSON 58.

Fractions — Continued.

ORAL EXERCISE.

How many? (Showing three circles.) George, write the figure for them on the board. Mary, cut one of these circles into halves. Do it carefully. How many halves have you? How many half-circles shall we have if we cut all the circles into halves? Cut them and see. How many halves are three times two halves? Which is more, three circles or six halves of a circle? John may go to the board and write in figures, after the 3, something that will say what he thinks. Is he right? What is the numerator of his fraction? Find the six things which that figure 6 stands for. In the same way, by figures, show how many halves equal four. Five. Two. Why is five equal to ten halves?

What have I? (Showing two and one fourth circles.) Irene, write the figures for them. Which figure stands for the whole circles? Which for the one fourth? I want to know how many *fourths there* are in two and one fourth circles taken together?

How many fourths in one circle? In two circles? You may cut them and show that there are eight fourths of a circle in two circles. How many fourths in all? Write figures to show that nine fourths equal two circles and a fourth. Write figures to show how many thirds two and two thirds are equal to. Complete in the same way what I now write on the board. (Writing—

$3=\frac{}{3}$ $5=\frac{}{3}$ $4=\frac{}{4}$ $8=\frac{}{5}$ etc.
$3\frac{1}{3}=$ $2\frac{3}{4}=$ $4\frac{1}{3}=$ $5\frac{1}{3}=$ etc.)

NOTE.—Rapid review work.

FOR STUDY.

I paid $4\frac{1}{2}$ dollars for a pair of shoes, paying 1 half-dollar at a time. How many half-dollars did I give?

A lady canned $3\frac{3}{4}$ gallons of peaches, using cans each holding $\frac{1}{4}$ of a gallon. How many cans did she use?

John cut 6 apples into thirds, and Robert cut 5 apples into fourths. Which had the more pieces? How many more? Why were there more pieces in the 5 apples than in the 6 apples?

A good walker can walk $4\frac{1}{3}$ miles an hour. How many thirds of a mile can he walk in an hour?

Walter's top has a string $1\frac{4}{6}$ feet long. How many inches long is it? How many sixths of a foot long would the string be if it were $2\frac{3}{6}$ feet long? How many inches?

A man paid $2\frac{3}{5}$ dollars for a hat. How many fifths of a dollar did the hat cost?

A box has $4\frac{1}{4}$ bushels of potatoes in it. How many fourths of a bushel are there in it? How many peck-baskets could be filled from the box?

A lady bought $2\frac{7}{8}$ pounds of steak. How many eighths of a pound did she buy? How many ounces?

Ella bought $3\frac{2}{3}$ yards of ribbon. How many thirds of a yard did she buy? How many feet?

How many sevenths of a week are there in $2\frac{3}{7}$ weeks? How many days?

Edgar picked $2\frac{1}{8}$ pecks of cherries. How many eighths of a peck did he pick? How many quarts?

Carl cut each of 3 oranges into fifths. How many boys could have 1 fifth?

A merchant bought $4\frac{3}{4}$ pounds of baking powder. How many cans, each containing $\frac{1}{4}$ of a pound, did he buy?

How many thirds in 1 orange? In 2 oranges? In $2\frac{1}{3}$ oranges? In $3\frac{2}{3}$ oranges?

How many fourths in 1 dollar? In $1\frac{1}{4}$ dollars? In 2 dollars? In $2\frac{3}{4}$ dollars?

How many sixths in 1 foot? In $1\frac{5}{6}$ feet? In 2 feet? In $2\frac{2}{6}$ feet?

How many eighths in 1 pound of steak? In $1\frac{1}{8}$ pounds? In $1\frac{5}{8}$ pounds? In 2 pounds?

How many halves in 6? 7? $7\frac{1}{2}$? 8?

How many thirds in 3? 4? $4\frac{1}{3}$? 5?

How many fourths in 3? $3\frac{2}{4}$? $3\frac{3}{4}$? 4?

How many fifths in 1? 2? $2\frac{2}{5}$? 3?

How many sixths in 2? $2\frac{3}{6}$? $2\frac{5}{6}$? 3?

How many sevenths in 1? $1\frac{2}{7}$? $1\frac{5}{7}$? 2?

How many eighths in $1\frac{3}{8}$? $1\frac{7}{8}$? 2? $2\frac{4}{8}$?

How many ninths in 1? $1\frac{1}{9}$? $1\frac{3}{9}$? $1\frac{5}{9}$?

How many tenths in 1? $1\frac{2}{10}$? $1\frac{5}{10}$? $1\frac{7}{10}$?

LESSON 59.

Fractions — Concluded.

ORAL EXERCISE.

How many? (Showing two thirds of an apple.) What kind? Roy, write the figures for this number. Lucy, take one half of them. How many has she taken? What part of an apple has Lucy? What is one half of two cents? What is one half of two thirds of an apple? Roy may write $\frac{1}{2}$ at the left of $\frac{2}{3}$ of an apple, and write the word *of* between them. Class, read what Roy has written. What part of an apple is one half of two thirds of an apple? Roy, at the right of $\frac{2}{3}$ write $\frac{1}{3}$ of an apple. Which means more, $\frac{1}{2}$ of $\frac{2}{3}$ of an apple or $\frac{1}{3}$ of an apple? Since they are equal to each other, put in the proper sign. Read what is written.

Horace, write $\frac{2}{3}$ of $\frac{3}{4}$ and put the sign equal after it. What is $\frac{1}{3}$ of 3 cents? What is one third of three apples? What is one third of three fourths of an apple? Then what is two thirds of three fourths of an apple? What is two thirds of any three? (Two of the same kind as the three.) Write $\frac{2}{4}$ after the sign. Read what is written. Show me that two thirds of three fourths are two fourths. Emma may do so with this apple. (First cut it into fourths.)

How many? (Showing one third of an apple.) What kind? I want to give Grace one half of it. Edith may write the figures for one half of one third of an apple. Have I two pieces this time, so that I can give Grace one of them? Then what can I do? (Divide the third into two equal parts, and give Grace one of them.) How many has Grace? What part of one third of an apple has she? Let us see what part of a whole apple Grace has. How many parts of this kind in one third of an apple? Then how many times two such parts in the whole apple? Ben, cut the other two thirds of the apple into halves and show me that there are six such parts in the whole apple. How many of the kind like the one Grace has make a whole apple? (Six.) Then what part of an apple has Grace? Edith, write $\frac{1}{6}$ of an apple just at the right of what you have written. Which means more, $\frac{1}{2}$ of $\frac{1}{3}$ or $\frac{1}{6}$? Make the sign to show that they are equal. Read what Edith has *written.*

Now what have I? (Showing two thirds of a circle.) I want to get one fourth of them. David may write one fourth of two thirds of a circle on the board. If I wanted one fourth of *one* third, how should I get it? Remember how we got one half of one third. How many such pieces do you get from *one* third? Then how many could you get from two thirds? How many of this kind in two thirds of a circle? (Eight.) How many are one fourth of eight? Now let us see what kind these two are. How many such parts did you have in one third? How many thirds are there in a whole one? Then how many times four such parts would there be in a whole one? Then how many like these two (holding them up) would make a whole circle? At the right of what he has written, David may write two twelfths of a circle. David, are the two fractions you have written equal to each other? Read them. If they are equal, put in the sign to say that they are equal. Class, read what David has written. What is one fourth of two thirds of a dollar? Finish this statement on the board: $\frac{1}{4}$ of $\frac{2}{3} =$. What does $\frac{2}{12} =$?

How many? (Showing two fifths of an apple.) What kind? I want to give Edward three fourths of these two fifths of an apple. Find what is one fourth of two fifths just in the same way that we found one fourth of two thirds. Into how many equal parts do you divide each of the two fifths? How many in one fifth? How many times four of them in two fifths? How many parts have you then? Show them to me. What is one fourth of these eight parts? Show me the two parts. How many such parts in one fifth of the apple? Then how many would there be in a whole apple? Why? What kind are these two? Then what is one fourth of two fifths of an apple? Say that in figures. ($\frac{1}{4}$ of $\frac{2}{5}$ of an apple $= \frac{2}{20}$ of an apple.) How many parts had you to take one fourth of? Show me the eight parts. How many fourths of two fifths do you want? Then how many of these twos must you take? Take them, Edward. How many have you, Edward? What kind are they? Then what is $\frac{3}{4}$ of $\frac{2}{5}$ of an apple? Lucy, say that on the board, in figures. Complete what I write now (writing $\frac{3}{4}$ of $\frac{2}{5} =$). What does $\frac{6}{20} =$?

Let us look at the answers to these last questions, as they are written on the board. Do you see that, in each writing, the numerator of the fraction on the right-hand of the sign is the number that we get when we take one of the first numerators as many times as there are ones in the other? (Let the children examine carefully and find that this is so.) Now look carefully and see that we can find the new denominator in the same way every time.

If I should give Ellen two fifths of two thirds of an apple, who can write the part of the whole apple that Ellen would get? George may write it. ("$\frac{2}{5}$ of $\frac{2}{3}=\frac{4}{15}$.") How did you get the 4? How did you get the 15? Ellen may take an apple and a knife and show that George's answer is right.

Complete these statements which I now write on the board. (Writing—

$\frac{1}{3}$ of $\frac{3}{5}=$ $\frac{3}{4}$ of $\frac{1}{3}=$ $\frac{2}{3}$ of $\frac{3}{5}=$ $\frac{2}{3}$ of $\frac{4}{5}=$ etc.)

NOTE.—It is important to lead the child to see that when he can divide the numerator of the second fraction by the denominator of the first—that is, when he can take directly one of the parts which the denominator calls for—he should do so. If this can not be done, he finds the products of the numerators and the denominators, respectively. See the two groups of exercises at the end of the study lesson.

FOR STUDY.

What part of an apple is $\frac{1}{2}$ of $\frac{2}{3}$ of an apple? $\frac{1}{2}$ of $\frac{6}{7}$ of an apple? $\frac{1}{2}$ of $\frac{10}{11}$ of an apple? $\frac{1}{2}$ of $\frac{14}{15}$ of an apple? How do you find these answers?

What part of a foot is $\frac{1}{3}$ of $\frac{6}{8}$ of a foot? $\frac{2}{3}$ of $\frac{6}{8}$ of a foot? $\frac{2}{3}$ of $\frac{9}{10}$ of a foot? $\frac{1}{4}$ of $\frac{8}{9}$ of a foot? $\frac{3}{4}$ of $\frac{8}{9}$ of a foot? How do you find these answers?

What part of a melon is $\frac{1}{5}$ of $\frac{10}{11}$ of a melon? $\frac{3}{5}$ of $\frac{10}{11}$ of a melon? $\frac{2}{5}$ of $\frac{15}{16}$ of a melon? How do you find these answers?

What part of a pound of butter is $\frac{1}{6}$ of $\frac{12}{16}$ of a pound? $\frac{3}{6}$ of $\frac{12}{16}$ of a pound? $\frac{5}{6}$ of $\frac{18}{20}$ of a pound? How do you find these answers?

What part of a bushel of potatoes is $\frac{1}{2}$ of $\frac{1}{2}$ of a bushel? $\frac{1}{2}$ of $\frac{3}{4}$ of a bushel? $\frac{2}{3}$ of $\frac{4}{5}$ of a bushel? $\frac{2}{3}$ of $\frac{2}{6}$ of a bushel? Can you find these answers as you found the others? How do you find them?

Frank gathered $\frac{3}{4}$ of a bushel of nuts, and gave Henry $\frac{1}{3}$ of them. What part of a bushel had Henry? How is the answer found?

A basket of peaches cost $\frac{4}{5}$ of a dollar. What did $\frac{1}{2}$ of a basket cost? How do you know?

Harry's father gave him $\frac{6}{8}$ of a dollar on his birthday. Harry spent $\frac{2}{3}$ of it for an arithmetic. What part of a dollar did the book cost? How do you find this answer?

What will $\frac{3}{4}$ of a pound of tea cost at $\frac{2}{3}$ of a dollar a pound? How is this answer found?

Mary had a half day to spend as she pleased. She spent $\frac{1}{3}$ of the time with a sick girl. What part of the day was she with the sick girl?

A man bought $\frac{4}{5}$ of a bushel of beans, and planted $\frac{3}{4}$ of them. What part of a bushel did he plant? Do you find this answer in the same way that you found the last one? How do you find it?

A farmer had $\frac{5}{6}$ of a cord of wood, and sold $\frac{2}{3}$ of it. What part of a cord did he sell? How is this answer found?

A man gave his daughter $\frac{3}{4}$ of a dollar a week to spend as she pleased. She gave $\frac{2}{3}$ of it to a poor woman. What part of a dollar did the poor woman receive?

What will $\frac{1}{2}$ of a pound of raisins cost at $\frac{1}{6}$ of a dollar a pound?

There were $\frac{6}{8}$ of a bushel of oats in a bag, and I fed my team $\frac{1}{2}$ of them. What part of a bushel did I feed my team? How can you find this answer?

Tom had $\frac{9}{10}$ of a dollar, and spent $\frac{1}{3}$ of it for a knife. What did the knife cost?

One pitcher holds $\frac{8}{9}$ of a quart, and a smaller one holds $\frac{3}{4}$ as much. How much does the smaller one hold? How do you know?

Fuel for a furnace costs $\frac{3}{8}$ of a dollar a day. What does it cost for $\frac{1}{2}$ a day?

Copy and complete as in the first two:

$\frac{1}{3}$ of $\frac{6}{7} = \frac{2}{7}$ $\frac{1}{5}$ of $\frac{10}{14} =$ $\frac{4}{7}$ of $\frac{7}{12} =$
$\frac{2}{3}$ of $\frac{6}{10} = \frac{4}{10}$ $\frac{4}{5}$ of $\frac{15}{16} =$ $\frac{6}{7}$ of $\frac{14}{15} =$
$\frac{1}{4}$ of $\frac{8}{9} =$ $\frac{3}{6}$ of $\frac{12}{16} =$ $\frac{5}{8}$ of $\frac{16}{17} =$
$\frac{3}{4}$ of $\frac{12}{13} =$ $\frac{5}{6}$ of $\frac{18}{20} =$ $\frac{4}{9}$ of $\frac{18}{20} =$

Copy and complete as in the first:

$\frac{2}{5}$ of $\frac{3}{4} = \frac{6}{20}$ $\frac{1}{4}$ of $\frac{1}{3} =$ $\frac{2}{3}$ of $\frac{4}{6} =$
$\frac{1}{2}$ of $\frac{1}{2} =$ $\frac{3}{4}$ of $\frac{2}{3} =$ $\frac{1}{2}$ of $\frac{7}{8} =$
$\frac{1}{2}$ of $\frac{3}{4} =$ $\frac{5}{6}$ of $\frac{2}{3} =$ $\frac{3}{4}$ of $\frac{1}{5} =$
$\frac{2}{3}$ of $\frac{1}{4} =$ $\frac{3}{5}$ of $\frac{3}{4} =$ $\frac{1}{2}$ of $\frac{7}{9} =$

LESSON 60.

Decimal Fractions.

ORAL EXERCISE.

NOTE.—In taking up decimal fractions, the pupil should be led to see that, in the decimal system, the position of a figure with respect to the decimal point does for it just what the denominator does for the numerator in a common fraction; that is, it tells how many of the kind which the figure represents are required to make a unit of the next higher order. Of course, in the decimal system, this required number is ten, invariably, whether the figure stands at the left or right of the decimal point. It should be impressed on the pupil that the decimal point is simply a starting point, a point from which to count the *place* of a figure. Its main purpose is *not* to separate whole numbers from decimal fractions. In fact, as a general rule, it is not best to separate them, but to read the whole expression as one number, and give the number the name of the right-hand figure. A mixed number in which the fraction is a common one is necessarily two numbers, as in $4\frac{2}{3}$ the numbers are four and two. But a mixed decimal is more naturally regarded as one number than as two numbers; for instance, 2.5 is generally better read as twenty-five tenths than as two and five tenths.

Look and see what I write on the board (writing $\frac{7}{8}$, $\frac{1}{8}$, $\frac{3}{8}$). What do you call such expressions? In such fractions, which figure shows how many are meant? (The figure above the line.) What do you call this figure? What tells you how many of this kind will make one of the next higher kind? Notice what I now write (writing $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$). Are these fractions, too? Read them. How many of the first kind will make one? How do you know? How many of the second kind? How many of the third kind? What tells you every time?

Now, I want to show you that there is another way in which these figures could be written so that you could know the same thing without any denominators. Read what I now write (writing 333). What kind is the first 3? The second 3? How do you know? The third 3? How do you know? You say the "second place" and the "third place." That is right; but what do we count from, so as to say second and third? That puzzles you a little. Well, I will write the figures this way. Notice (writing 333.). Now, what can you count from? Yes, that is what the little dot is there for, just to count from. We do not always write it, but we always count just as if it were written. Such a little dot is called a *decimal point*. I want you to remember the name of it.

Now, if I write a 1 in the second place to the left of the decimal point, in this way (10.), what does the 1 stand for? Yes, you learned that a good while ago. And what does the zero show? Now, if I should erase the zero and put the 1 in the first place, (suiting the action to the word), what does the 1 stand for? It means what part of what it meant before? If it meant ten dollars before, what does it mean now? What part of ten dollars is one dollar? What is one tenth of one dollar? Where, then, can I put the 1 so that it will stand for one dime? (One place farther to the right.) That puts it on the other side of the decimal point, but no matter for that. Now, what is one tenth of a dime? Then where should I put the 1 to make it stand for one cent? (Move it another place to the right.) Then it will be in the second place to the right of the decimal point; but is there any*thing to stand* in the first place so as to show that the 1 is in the second place? Who remembers what we did to mark the first place on the other side of the point, when we put 1 in the second place to stand for ten? Well, we will do the same thing now; we will put a zero in the first place in this way (.01). How many cents make a dollar? Then what part of a dollar is one cent? (One hundredth.) Yes, and that is what our figure stands for now. What do I show you? (Show a silver dollar, a dime, and a cent.) How many dollars? Well, we will write the figure 1 for it in the ones' place, just at the left of the decimal point, in this way ($1.). You see I have put a little sign before the 1, and this sign always shows that the figures mean dollars. Now where shall I write a 1 to stand for one dime, or one tenth of one dollar? (Just at the right of the point.) And where shall I put a 1 to stand for one cent? Yes, and so our writing for these coins will stand in this way ($1.11). Mary, take the coins and lay them on the table just as these figures stand for them.

Now, see if you can read the figures I write. (1.2.) How many? What kind is the twelve? (Tenths.) (1.25.) How many? What is the hundred twenty-five? Point to the figure that stands for tenths. How many tenths? Point to the figure that stands for hundredths. How many hundredths? How many hundredths in all? What does this stand for (.3)? And this (.03)? And this (.003)? Do you see, then, that we have found a way to write three tenths and three hundredths and three thousandths without any denominators? Now we will write them all together in this way (.333). Read. Why do we not have any zeros now? (Because we do not have to mark any place that has no figure in it.)

Now, see if you can read these figures as I write them: .4, .07, 4.5, 12.5, .222, .303, etc.

All go to the board. Write six tenths. Twenty-four tenths. Twenty-four hundredths. Twenty-four thousandths, etc.

All write two hundred sixteen. Put the point so that it will mean two hundred sixteen tenths. Now move all the figures one place to the right. What is the name of the two hundred sixteen now? Move all the figures three places to the left. Now you have two hundred sixteen what? In what place is the 6? (Tens' place.) Then what must you put in the ones' place? How many has our number been all the time? (Two hundred sixteen.) *What has changed* when we moved the figures? (The name.)

All write these fractions without denominators. (Writing $\frac{4}{10}$, $\frac{8}{1000}$, $\frac{124}{100}$, $\frac{95}{10}$, etc.) Now write these decimals with denominators. (Writing .7, .15, 1.8, .025, etc.) When you write denominators what becomes of the decimal point? (We erase it.) How many zeros do we put at the right of 1 in the denominator? (Just as many as the number of places at the right of the point.)

FOR STUDY.

Read the following:

$1000.	$2000.	$7000.
$100.	$200.	$700.
$10.	$20.	$70.
$1.	$2.	$7.
$.1	$.2	$.7
$.01	$.02	$.07
$.001	$.002	$.007

$1300.	$2570.	$7050.
$130.	$257.	$705.
$13.	$25.7	$70.5
$1.3	$2.57	$7.05
$.13	$.257	$.705
$.013		

8 gallons.	15 feet.	236 pounds.
.8 of a gallon.	1.5 feet.	23.6 pounds.
.08 of a gallon.	.15 of a foot.	2.36 pounds.
.008 of a gallon.	.015 of a foot.	.236 of a pound.

Read the following decimals. In each case tell:

First. How many are given.

Second. What kind each is, and what shows it.

Third. How many of the kind given make a larger one:

.8 .07 .06 .009 .004 .04

Model for the first: "Eight are given. They re tenths, because the figure stands in the first lace to the right of the point. Ten like the ight would make a whole unit."

Write the following fractions without denomiators:

$\frac{4}{10}$
$\frac{9}{10}$
$\frac{6}{100}$
$\frac{43}{100}$
$\frac{72}{100}$
$\frac{1}{1000}$
$\frac{14}{1000}$
$\frac{72}{1000}$
$\frac{239}{1000}$
$\frac{58}{10}$
$\frac{137}{10}$
$\frac{1001}{1000}$
$\frac{608}{100}$
$\frac{705}{1000}$

Five tenths.
Nine tenths.
Seven hundredths.
Thirteen hundredths.
Twenty-three hundredths.
Nine thousandths.
Nineteen thousandths.
One hundred sixteen thousandths.
Two hundred one thousandths.
Two hundred seven thousandths.
Twenty-four tenths.
One hundred twenty-nine tenths.
Twenty-seven thousandths.
Six hundred thirty-five hundredths.

Change the following decimals to fractions with enominators:

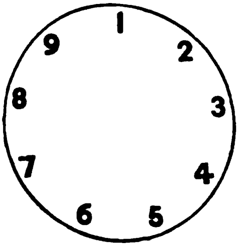
.5
.21
6.07
2.08
.08
6.2
3.015
.063
.18
.37
7.4
9.02
2.5
3.12
.003
8.001

LESSON 61.

Addition.

ORAL EXERCISE.

NOTE.—Turn back to the tables of 45 combinations. Have them written on the board. Practice on the combinations in their four forms, for three minutes. Have the splints and rubber bands ready for use. Put on the board the nine figures around the circumference of a circle, as shown. Use this arrangement for a little while every day, in drilling on the combinations. Write a figure in the center, then point rapidly to the figures in the circumference, letting the class add their values to the value of the one written in the center. Change the figure in the center, and repeat, etc.



NOTE.—DRILLS IN ADDITION.—A great deal of varied practice is necessary in order to acquire facility and accuracy in addition. The class should exercise a few minutes each day in adding from oral dictation. Let the teacher call off numbers to be added: "Six, five, nine, four," etc. As soon as his voice falls, the answer should be given simultaneously. The numbers should be given as fast as the class can follow; only a few easy numbers at first; more numbers, and at a more rapid rate, as the class gains in power. Vigorous, persistent work of this sort will achieve results astonishing to one who has never tried the experiment. Neglect of sufficient practice in these fundamental operations is one of the most harmful errors in the teaching of arithmetic.

Another good exercise is to send the class to the board and let them write, from the teacher's dictation, figures in a vertical column. As soon as the teacher's voice falls, let each pupil draw a horizontal line under his column and proceed to add as quickly as possible. As soon as he has a result let him write it, and turn at once from the board. Here are specimen columns for such an exercise:

(1)	(2)	(3)
4	5	2
5	7	9
9	6	7
7	4	6
6	9	5
8	3	4
3	8	7
42	42	40 etc.

NOTE.—Let the child be taught to avoid saying, "Three and eight are eleven, and six are seventeen," etc., in adding the first column. Train him to say at once, "Three, eleven, seventeen, twenty-four," etc. If the addition of the next number does not give another ten, let him speak only the units in the next result; for instance, let him finish the addition of column (1) by saying, "Thirty-three, eight, forty-two." When the next two or three numbers amount to exactly ten, train him to observe the fact and to add the ten at once—as in column (2) let him say, "Eight, eleven, twenty, thirty, seven, forty-two." Teach him to use as few words as possible. In all cases, let him form the habit of proving the correctness of his result by immediately adding in the opposite direction. If the columns are of any considerable length, it is very improbable that the two results will be the same unless both are correct.

Fred, take three bunches of hundreds, four of tens, and three loose splints, and lay them on the table in the right order. How many splints are there in all? Willie may write the figures for them on the board. Now lay one hundred, two tens, and five loose splints close to the others, and nearer to you, Fred. How many splints? Willie may write the figures for them under the figures for the first group, taking care to write the figures for the loose splints in a column, the figures for tens in another column, and the figures for hundreds in a column. Now lay four hundreds, three tens, and one loose splint by the other two groups, still nearer to you, Fred. How many splints in this group? Willie may write the figures for them under the figures for the others. Draw a horizontal line beneath the figures.

Where are the three tens? Where is the figure for them? Where are the three hundreds? Where is the figure for them? Find the splints for which this figure (pointing to the 5 units) stands. Now, see that your splints lie in just the same order as the figures that stand for them.

We want to find how many splints in all there are in these three groups. Where shall we begin putting them together, Helen? Why do you think it is better to begin with the loose sticks and not with the bunches? (Helen should say, "There may be ones enough to make a bunch of ten; and if there are we ought to have the bunch ready to put with the other bunches of ten when we put them together.") You may put the ones together, Helen. How many are three and five and one? Don't count them. Lay them to one side, and write the figure 9 for them *under the figures* on the board which stand for ones.

What shall we put together next, Hubert? Why should the tens be put together next? (Hubert should say, "There may be tens enough to make a bunch of a hundred; and if there are, we ought to have the bunch ready to put with the other bunches of hundreds when we put them together.") You may put them together, Hubert. How many tens are four and two and three tens? Lay them in the right place near the ones, and make the figure 9 for them in the proper place. Now put the hundreds together. How many are three and one and four hundreds? Lay them near the tens, and make the figure 8 for them in the proper place. How many splints are there in all? Where are the figures for them?

We call this work adding, or addition. You have added a good deal before, but not such large numbers. The number that we find, which is equal to the other numbers counted together, we call their *sum*. Remember this word, and remember what it means.

Hugh may lay two hundreds, seven tens, and six ones in the right order on the table. Nellie, write the figures for them on the board. Near the first group, in the proper place, lay three hundreds, nine tens, and seven ones. How many? Write the figures for them, Nellie, beneath the figures for the other number. Near the two groups Hugh may lay six hundreds, three tens, and five ones. How many? Write the figures for them, Nellie, in the proper place, and draw a line beneath them. Where are the numbers to be added? Where are the figures for them? Are the figures in the same order as the numbers? Are we going to add the figures or the numbers? Where shall we begin? Why? Laura may put the ones together. How many are there? Don't count them. (Eighteen.) What shall be done with them? (Bunch ten of them.) Do so, Laura. What have you now? (One ten and eight loose ones.) What shall be done with the ones? (Lay them to one side.) Write the figure 8 for them in the proper place. What shall be done with the one ten? Robert may put the tens together. How many? (Twenty tens.) What shall be done with them? How many bunches of one hundred do twenty tens make? (Just two.) What shall be done with them? Have we any figure to write in tens' place this time? (Yes.) What is it? (Zero.) Why write a zero? (So that we may know that the next place is for hundreds.) What is the next thing to do, Laura? (Put the hundreds together.) Do so. How many? (Thirteen.) *What shall be* done with them? (Bunch ten of them.) Do so, Robert. Now what have you? (One thousand and three hundreds.) What shall be done with them? (Lay them in the proper place by the ones.) What figures do we write? Where shall the 3 be written? Why? Where shall the 1 be written? Why? Where is the *sum* of all the numbers? Where are the figures for the sum? Read them. Did we add the figures or the numbers? (The numbers.) Yes, and the work with figures just "stood for" the work with numbers. Work with figures always stands for some work that may be done with things.

All may go to the board. We will do the work with figures alone and find how many splints there are in three groups, without actually putting the splints together: One contains two hundred seventy-eight splints; another, three hundred fifty-six splints; the other, six hundred ninety splints. Write the numbers and find out how many splints there will be in their sum. See who will get a correct answer first.

FOR STUDY.

Here are some tables that you had a good while ago. If you have not learned them thoroughly, you should do it now, for you will need to use them every day, and many times a day:

TABLES FOR ADDITION AND SUBTRACTION.

1+1=2
1+2=3
1+3=4
1+4=5
1+5=6
1+6=7
1+7=8
1+8=9
1+9=10

2+2=4
2+3=5
2+4=6
2+5=7
2+6=8
2+7=9
2+8=10

3+3=6
3+4=7
3+5=8
3+6=9
3+7=10

4+4=8
4+5=9
4+6=10
5+5=10

6+5=11
6+6=12
6+7=13
6+8=14
6+9=15

7+4=11
7+5=12
7+7=14
7+8=15
7+9=16

8+3=11
8+4=12
8+5=13
8+8=16
8+9=17

9+2=11
9+3=12
9+4=13
9+5=14
9+9=18

1. Lyman weighs 98 pounds and Tom weighs 95 pounds. How many pounds do both together weigh? Write the figures and find the sum.

2. Hattie solved 14 problems on Monday, 18 on Tuesday, 12 on Wednesday, 15 on Thursday, and 17 on Friday. How many did she solve in the 5 days?

3. A farmer sold three loads of oats. The first load contained 48 bushels; the second, 53 and the third, 46. How many bushels did he sell?

4. Albert's reader cost 75 cents; his geography, 89 cents; his arithmetic, 47 cents; and his slate, 23 cents. How many cents did he pay for all? How many dollars and cents did they cost?

5. A merchant bought five rolls of cloth containing respectively, 57 yards, 29 yards, 63 yards, 48 yards, and 72 yards. How many yards in all?

6. A milkman sold 76 quarts of milk on Monday, 68 on Tuesday, 87 on Wednesday, and 79 on Thursday. How many quarts of milk did he sell in all?

7. A steamship sailed 246 miles the first day, 217 miles the second, 265 miles the third. How many miles did it sail in the three days?

8. A man had 352 sheep. How many had he after he bought 437 more?

9. A miller sold 278 barrels of flour to one man, 563 barrels to a second, and 236 to a third. How many did he sell to all?

10. A farmer has 638 bushels of wheat in one bin, 479 bushels in a second, and 387 bushels in a third. How many bushels of wheat has he?

11. A man raised 134 bushels of potatoes one year; the next year he raised 45 bushels more than the first year; and the third year he raised 26 bushels more than the second year. How many bushels did he raise in the three years?

12. A owns a farm of 472 acres; B, one of 513 acres; and C, one of 396 acres. How many acres do they all own?

13. A man bought four loads of hay. The first weighed 2,030 pounds; the second, 2,187 pounds; the third, 2,234 pounds; and the fourth, 2,316 pounds. How many pounds did the four loads weigh? How many tons?

14. A man bought fruit for $637.75, and sold it so as to gain $179.87. How many dollars did he receive for it?

15. A man paid $4.65 for a barrel of flour, $32.35 for a barrel of sugar, $33.25 for a bag of coffee, and $13.85 for two tons of coal. How many dollars did he pay for all?

16. A man earned $6.65 one week, $7.15 the second, $6.40 the third, $8 the fourth week, and $7.75 the fifth week. How many dollars did he earn in the five weeks?

17. A family used .78 of a ton of coal in December, .825 of a ton in January, .675 of a ton in February, .625 of a ton in March, and .5 of a ton in April. How many tons did they use in the five months?

18. A lady spent $7.35 for a hat, $26 for a dress, $18.75 for a cloak, $2.15 for gloves, and had $25.75 left. How many dollars had she at first?

Find the sums of these **numbers:**

(1)	(2)	(3)	(4)	(5)	(6)
705	7.15	846	67.5	8.47	.387
820	.35	793	80.	9.04	.296
356	2.04	217	74.86	6.72	.704
			93.25	5.5	.583

Copy and find the sums:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
54	12	64	32	97	387	307
46	34	92	45	84	468	596
33	56	37	96	44	956	438
26	23	41	18	26	736	943
52	45	39	72	14		
27	56	73	14	57		

(8)	(9)	(10)	(11)	(12)
659	476	$43.25	$58.125	$28.50
278	582	4.76	3.065	.625
765	174	13.62	5.50	50.
294	785	27.18	16.666	4.444

LESSON 62.

Subtraction.

ORAL EXERCISE.

Arthur may lay five hundreds, three tens, and eight ones of splints in the right order on the table. Fannie, write the figures for them on the board. Now, we are going to take two hundreds, three tens, and six ones of splints out of this group, to find how many splints will be left. Fannie, write the figures for the last number under the figures you have already written. Draw a straight line beneath the figures. How many splints in all are we going to take out of this group? Where are they *now?* Where are the figures that stand for them? How many loose splints do *we wish to take* away? Where shall we find them? Where is the figure that stands for them? Six from eight will leave how many? Lay the six by themselves. How many loose splints are left? Lay the two splints in another place. Where shall we write the 2 to stand for them? Fannie, write it. How many tens do we want to take away? Three tens from three tens leave how many? Arthur, lay the three tens with the six loose sticks. How many are left? Have we any figure to write? Why should we write a zero? (To show that the next figure will stand in hundreds' place.) Write it, Fannie. How many hundreds do we want to take out? Two hundreds from five hundreds leave how many? Arthur, lay the two hundreds with the three tens and six ones. How many hundreds are left? Lay them with the two loose splints. Where shall we write the 3 to stand for the hundreds that are left? Write it, Fannie.

How many splints were there at first? Where are the figures for them? How many have we taken away? Where are they? Where are the figures that stand for them? How many splints are left? Where are they? Where are the figures for them? How can we make again the number we had at first? Charles may put them together just as we did in addition, and Fannie may write the figures on the board. The others will see whether Charles and Fannie do their work right.

Now for a different example:

Harold may lay six hundreds, four tens, and five loose splints in right order on the table. Ella may write the figures for them on the board. Now we are going to take two hundreds, four tens, and seven ones out of this group, to find how many splints will be left. Ella may write the figures for the number we are going to take away, in the proper place, and draw a line beneath. How many splints are there in the group? Where are the figures for them? How many splints are we going to take away? Where are they now? Where are the figures that stand for them? How many loose splints do we want to take away? (Seven.) But we have only five; what shall we do? (Some one will doubtless say, "Untie one bundle of tens.") Do so, Harold. What shall we do with the ten loose ones? (Put them with the five loose splints.) How many loose splints have we now? (Fifteen.) Seven from fifteen leave how many? Harold, take the seven and lay them by themselves. How many loose splints are left? Lay the eight splints in another place. Write the figure for them, Ella, in the proper place. How many tens do we want to take away? How many tens are there? What shall we do, Jennie? (Untie one bunch of hundreds.) Do so, Jennie. How many. tens now? (Thirteen.) Four from thirteen leave how many? Harold, take out the four tens, and lay them with the seven loose splints. How many tens are left? Lay them with the eight loose splints, and write the figure for them in the proper place, Ella. How many hundreds do we want to take away? How many have we now? Two from five leave how many? Take the two and lay them with the others that we took out. How many are left? Lay the three hundreds with the others that were left. Write the figure for them, Ella.

How many splints were there at first? Where are the figures for them? How many have we taken away? Where are they? Where are the figures for them? How many splints are left? Where are they? Where are the figures for them?

Now let us take one example just with figures, not using the splints. Suppose we have eight hundred six for our larger number. Peter, write the figures on the board. In subtraction, the larger number is always called the *minuend.* (I will write the word on the board. Remember the word, and remember how it is spelled.) Now, we want to take away from the number, two hundred forty-nine. Write the figures for it in the right place, Peter, and draw the line. We call the smaller number the *subtrahend.* This is the way to spell it (writing the word). How many ones do we want to take away? How many have we? What shall we do? (Some one will say, "We have no tens.") Well, suppose we had the splints; how could we get some tens? (Undo a bunch of one hundred.) Yes, and how many hundreds should we have left? (Seven.) How many tens should we get from the bunch of one hundred? (Ten.) Now, how can we get some loose sticks? (Undo a bunch of ten.) How many bunches of ten will be left? (Nine.) How many loose sticks shall we have now? Nine from sixteen leaves how many? Write the figure 7 in the right place, to show how many ones are left. How many tens had we? Four from nine leaves how many? Write the figure 5 in the right place. How many hundreds had we? Two from seven leaves how many? Write the 5 in hundreds' place. Then, how many will be left if we take two hundred forty-nine out of eight hundred six? Point to the minuend. Point to the subtrahend. *Point to the* figures that show how many are left. The number that is left is always called the *difference*, or the *remainder*. I will write the words under the other words (writing them).

If we should add the difference and the subtrahend together, what ought the sum to be equal to? (The minuend.) Yes, that is easy to see. We took the subtrahend out of the minuend, and have the remainder left; then if we put the subtrahend and remainder together, we ought to have the minuend. In this way we can prove our work to see that we have made no mistake. You may do it just as we did in addition.

FOR STUDY.

1. Ella had 85 cents and spent 23 cents for ribbon. How many cents had she left? Write the figures and find the answer.

2. Hugh weighs 96 pounds and Warren weighs 78 pounds. How many more pounds does Hugh weigh than Warren?

3. Two boys picked 84 quarts of berries. How many did the second boy pick if the first boy picked 48 quarts?

4. A horse and a cow together cost $212. If the cow cost $45, what was the cost of the horse?

5. A man has 176 sheep. How many must he buy to have 341?

6. A man bought a horse for $385, and sold it for $457. How many dollars did he gain?

7. Mr. Smith bought 720 feet of lumber, and used 573 feet. How many feet were left?

8. Daniel Webster was born in 1782, and he died in 1852. How old was he at his death?

9. A miller had on hand 378 barrels of flour; he received an order for 650 barrels. How many more barrels must he make before he can fill the order?

10. Elmer wrote 532 words on Monday and 396 on Tuesday. How many more did he write on Monday than on Tuesday?

11. An army went into battle with 5,750 men, and came out with only 3,874 men. How many were missing?

12. How many years since Columbus discovered America in 1492?

13. Mr. Brown's yearly income is $5,067; his expenses are $4,093. How many dollars does he save?

14. A tank which holds 375 gallons of water has only 95 gallons in it. How many more gallons must be pumped into the tank to fill it?

15. The Pilgrims landed at Plymouth in 1620. How many years ago was that?

16. A man got 450 bricks to build a chimney. The bricklayer said he should need 600 bricks. How many more must he get?

17. In a week one steamer sailed 2,065 miles while another sailed 1,986 miles. How many more miles did one sail than the other?

18. A man bought a house for $4,635 and sold it at the end of six months for $5,470. How many dollars did he gain?

19. A man had $39.65. How much more must he have to buy a horse for $145.50?

20. A grocer made a profit of $65.45 on sugar which he sold for $736.12. How many dollars did the sugar cost?

21. What sum of money must be added to $562.72 to make $843.50?

22. A man has $896.25 in the savings bank and $387.40 less in his safe. How many dollars has he in his safe? How many dollars has he in both places?

23. If a man's annual income is $2,250 and he spends $1,834.23, how many dollars does he save each year?

Copy these figures, find the differences, and prove the results:

(1)	(2)	(3)	(4)	(5)	(6)	(7)
79	96	85	76	89	97	77
34	64	50	52	35	24	23

(8)	(9)	(10)	(11)	(12)	(13)	(14)
98	83	87	42	54	65	68
43	23	42	14	26	28	39

(15)	(16)	(17)	(18)	(19)	(20)	(21)
72	76	83	87	90	96	546
36	29	48	39	43	57	257

(22)	(23)	(24)	(25)	(26)	(27)	(28)
631	408	763	600	732	985	821
272	239	347	379	456	278	736

(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)
527	980	772	967	922	736	618	813
318	142	624	209	666	59	40	307

(37)	(38)	(39)	(40)	(41)	(42)	(43)
$57.85	$6.25	$10.00	$4.50	$175.00	$7.375	$8.75
39.27	1.75	3.75	.85	87.50	2.625	1.875

LESSON 63.

Addition and Subtraction—Continued.

ORAL EXERCISE.

NOTE.—Save time by having the material needed for this lesson ready before the class is called. Present the work in such a way as to show that addition is the same thing, and is performed in essentially the same way, in Simple Numbers, including Decimals and Federal Money, and in fractions and compound numbers. The same thing is true of subtraction. The only difference is that in Simple Numbers, Decimals, and Federal Money, ten units of one kind always make one of the next higher order; while, in the other cases, the number of one kind required to make one of the next higher is variable. This is the difference, and the only difference, between simple and compound numbers.

Give several exercises like the first one below, with such numbers as two fourths and one fourth, two sixths and one sixth, two sixths and four sixths, etc. When the sum amounts to a whole one or more, change to a whole or mixed number.

How many? (Showing one third of a circle.) —— what? How many in this hand? (Showing another third.) How many now? (Putting both into one hand.) —— what? Ethel may write it on the board. James may tell a story about one third and one third.

How many? (Showing one half of a circle.) How many in this hand? (Showing one fourth of a circle.) Are these two alike? Can we add them as they are? (No.) What must we do to them before we put them together? (Make them alike.) How would you make them alike? (Cut the one half into two equal parts and they will be like the one fourth.) We will do it. How many fourths are there in one half? Two fourths and one fourth are how many? —— what?

How many? (Showing one third of a circle.) What have I in this hand? (Showing one half of a circle.) Can they be made alike? (Yes, they can.) Both of them must be cut. Does any one see how they may be cut so as to make all parts equal in size? Well, we will cut the one third into two equal parts. Now, how many like this will make a circle? What is their name? (Sixths.) Can we do anything to the one half to make sixths of it? (Cut it into three equal parts.) Agnes may write on the board, $\frac{1}{3} = \frac{\ }{6}$. Below write $\frac{1}{2} = \frac{\ }{6}$. Fill the blanks. Now *write* $\frac{2}{6} + \frac{3}{6} =$.

What have I? (Showing $2\frac{1}{4}$ circles.) Ellen may write the figures for them on the board. I will lay them on the table. Now what have I? (Showing $4\frac{3}{4}$ circles.) Write the figures for them beneath the $2\frac{1}{4}$ for adding, Ellen. I will lay them near the $2\frac{1}{4}$ circles to add them. What have I now? (Showing $5\frac{2}{4}$ circles.) Write the figures for them, Ellen, and draw a line underneath. I will lay them near the others.

What would you do first, Warren, to find how many circles all these will make? Why would you put the parts of circles together first? (There may be enough of them to make one or more whole circles.) Put them together, Warren. How many fourths of a circle? How many whole circles and how many fourths over are there in six fourths of a circle? Write figures for the two fourths in the proper place. What will you do with the one circle? How do you know that you must "carry" one for four? How many circles in all? Write twelve in the proper place. Read the *sum*. You may go to the board and we will try one or two examples like these, using nothing but the figures. (Give an example or two.)

How many? (Showing two thirds of a circle.) —— what? How many do I take away? (Taking one away.) How many are left? Two thirds less one third are how many thirds? Fred may write it on the board.

(Give several exercises like this.)

What have I? (Showing two and one fourth circles.) I want to give Nancy three fourths of a circle, and I have but one fourth. What can I do? (Cut one of the circles into fourths.) Do so, Willie. How many fourths are there now? Take out three fourths. How many are left? Two and one fourth circles less three fourths of a circle are how many circles? Write it on the board, Willie. John may tell a story about two and one fourth less three fourths.

You may go to the board and we will try one or two such examples with figures only. (Giving one or two similar examples.)

I have three fourths of a dollar, and six eighths of a dollar, and one half a dollar, and I want to find how many dollars I have in all. Silas may write the figures for these parts of a dollar on the board as you have learned to write figures when you are going to add. Now look at these figures carefully. How much does the first fraction lack of a whole dollar? (One fourth.) I wonder if we can get one fourth out of one of the other fractions to put with these three fourths to make a whole dollar? (Some one may say, "Take two of the six eighths; they will make one fourth.") Now we have one dollar, and how many eighths are left? What are four eighths equal to? What shall we do with this one half? (Put it with the other half.) Now how many dollars have we? Have we any fraction left?

Children, when you have fractions to add, I want you always to look very sharply to see if you can not make whole ones in some such way as we did this time. You can not do it always, but quite often you can. See how many can do it with the examples in this lesson in your book.

Exercises for rapid work:

NOTE.—Give these exercises orally and as rapidly as the class can follow them.

$\frac{2}{4} + \frac{1}{4} =$
$\frac{4}{5} - \frac{1}{5} =$
$\frac{6}{7} + \frac{3}{7} =$
$\frac{7}{8} - \frac{3}{8} =$
$\frac{3}{5} + \frac{4}{5} =$
$\frac{9}{10} - \frac{3}{10} =$

$\frac{1}{6} + \frac{3}{6} =$
$\frac{6}{7} - \frac{2}{7} =$
$\frac{4}{8} - \frac{3}{8} =$
$\frac{10}{11} - \frac{4}{11} =$
$\frac{5}{12} + \frac{8}{12} =$
$\frac{12}{15} - \frac{8}{15} =$

$\frac{2}{6} + \frac{4}{6} =$
$\frac{8}{9} - \frac{3}{9} =$
$\frac{6}{11} + \frac{7}{11} =$
$1 - \frac{7}{13} =$
$1 - \frac{8}{14} =$
$\frac{8}{16} + \frac{9}{16} =$

$\frac{13}{18} - \frac{9}{18} =$
$\frac{10}{19} + \frac{9}{19} =$
$1 - \frac{18}{20} =$
$\frac{8}{17} + \frac{9}{17} =$
$\frac{18}{19} - \frac{9}{19} =$
$\frac{8}{20} + \frac{8}{20} =$ etc.

FOR STUDY.

1. Robert caught a trout weighing $\frac{3}{5}$ of a pound, and Henry caught one weighing $\frac{4}{5}$ of a pound. How many fifths of a pound did both trout weigh together? How many pounds?

2. From a stick $\frac{10}{12}$ of a foot long, George cut off $\frac{4}{12}$ of a foot. How many twelfths of a foot were left? How many inches? What part of a foot?

3. Susan picked $\frac{1}{4}$ of a bushel of cherries, Jane picked $\frac{3}{4}$ of a bushel, and Mary $\frac{1}{2}$ of a bushel. How many bushels did all three pick?

4. If a ton of hay is bought for \$8$\frac{1}{4}$ and sold for \$9$\frac{3}{4}$, how many dollars are gained? (No figures.)

5. A dealer sold $8\frac{3}{4}$ tons of coal to one man and $6\frac{5}{8}$ tons to another man. How many tons did he sell to both men? (No figures.)

6. Mr. Jones paid $\$3\frac{3}{10}$ for wood, $\$7\frac{7}{10}$ for flour, $\$14\frac{9}{10}$ for sugar. How many dollars for all?

7. A box of soap weighed $75\frac{3}{8}$ pounds, and the box alone weighed $3\frac{5}{8}$ pounds. How many pounds did the soap weigh?

8. A farmer has three stacks of hay. The first contains $3\frac{4}{7}$ tons, the second contains 4 tons, and the third contains $4\frac{3}{7}$ tons. How many tons in all three stacks? (No figures.)

9. A boy and his bicycle together weigh $94\frac{3}{16}$ pounds; the boy weighs $73\frac{5}{16}$ pounds. How many pounds does the bicycle weigh? Write the figures and subtract.

10. A man bought a cow for $\$44\frac{3}{4}$ and a sheep for $\$9\frac{1}{2}$. How many more dollars did he pay for the cow than for the sheep?

11. James picked $5\frac{3}{8}$ bushels of apples from one tree, $7\frac{1}{2}$ bushels from another, and $6\frac{5}{8}$ bushels from another. How many bushels from all?

12. A piece of cloth measured $12\frac{1}{3}$ yards before sponging and $10\frac{2}{3}$ yards after sponging. How many yards did it shrink?

13. A grain dealer bought wheat at $60\frac{7}{8}$ cents a bushel and sold it at $61\frac{5}{8}$ cents a bushel. How much did he gain on each bushel?

14. Mary learned her spelling lesson in $\frac{7}{15}$ of an hour, her reading lesson in $\frac{10}{15}$ of an hour, and her arithmetic lesson in $1\frac{4}{15}$ hours. How many hours did it take her to learn all three?

15. A man owned 14 acres of land and sold $4\frac{5}{6}$ acres. How many acres had he left?

16. Lewis had \$$2\frac{1}{4}$; he earned \$$1\frac{1}{4}$ more, and then paid \$$1\frac{3}{4}$ for a hat. How many dollars had he left? (No figures.)

17. A merchant had a roll of cloth containing $56\frac{1}{3}$ yards. He sold to one lady $7\frac{2}{3}$ yards, and to another $9\frac{2}{3}$ yards. How many yards were left?

18. Elmer earned \$$57\frac{1}{4}$ one summer vacation. This was \$$14\frac{3}{4}$ more than his brother earned. How many dollars did both boys earn?

19. A grocer bought $8\frac{4}{16}$ pounds of butter. He sold $2\frac{8}{16}$ pounds to one customer, and $3\frac{12}{16}$ pounds to another. How many pounds had he left?

20. Mr. Adams owned a lot containing $3\frac{2}{5}$ acres. He bought one containing $2\frac{4}{5}$ acres. He then sold $1\frac{3}{5}$ acres. How many had he left? (No figures.)

21. Nellie is $4\frac{3}{6}$ years old, Sarah is $1\frac{5}{6}$ years older than Nellie, and Walter is $3\frac{1}{6}$ years older than Sarah; how old is Walter? How many years older is Walter than Nellie?

Copy and find the sums:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$5\frac{2}{5}$	$4\frac{2}{7}$	$3\frac{5}{9}$	$2\frac{1}{4}$	$3\frac{1}{4}$	$1\frac{1}{3}$	$6\frac{5}{12}$	$2\frac{5}{14}$
$1\frac{1}{5}$	$6\frac{5}{7}$	$2\frac{3}{9}$	$8\frac{1}{2}$	$2\frac{3}{8}$	$2\frac{1}{2}$	$3\frac{4}{12}$	6
$2\frac{4}{5}$	$3\frac{1}{7}$	$\frac{7}{9}$	$5\frac{3}{4}$	$3\frac{2}{4}$	$3\frac{1}{3}$	$4\frac{7}{12}$	$4\frac{3}{14}$
$7\frac{3}{5}$	$5\frac{4}{7}$	$4\frac{2}{9}$	$3\frac{1}{2}$	$1\frac{1}{8}$	$4\frac{2}{3}$	$2\frac{8}{12}$	$7\frac{2}{14}$

Copy and find the differences:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$8\frac{4}{5}$	$9\frac{5}{6}$	$12\frac{7}{8}$	$10\frac{1}{10}$	$13\frac{4}{11}$	6	8	$9\frac{4}{15}$
$3\frac{1}{5}$	$2\frac{3}{6}$	$5\frac{4}{8}$	$4\frac{3}{10}$	$6\frac{8}{11}$	$4\frac{5}{8}$	$2\frac{7}{16}$	$2\frac{7}{15}$

LESSON 64.

Addition and Subtraction—Continued.

ORAL EXERCISE.

Here are three strings. I'll give one to Harry, one to Edgar, and the other to Walter. We want to find how far they will reach if laid end to end on the floor. Each of you may measure the string he has. How long is your string, Harry? (2 yds. 1 ft. 5 in.) Ollie may write the figures for the length of Harry's string on the board. Above the 2 write *yd.*, above the 1 write *ft.*, and above the 5 write *in.* Read what you have written. How long is your string, Edgar? (1 yd. 2 ft. 11 in.) Ollie, write the figures for the length of Edgar's string beneath the other figures, for adding. How long is your string, Walter? (2 yds. 2 ft. 3 in.) Write the figures, Ollie. Draw a line beneath the figures. What does the left-hand column of figures stand for? The right-hand column? The middle column? Where shall we begin to add? Why begin with inches? (There may be inches enough in all to make a foot; and if there are, we want to put it with the feet for which the figures in the next column stand.) How many are 5 inches, 11 inches, and 3 inches? How many inches in one foot? How many feet in nineteen inches? How many inches over? What figure do we write? Write it, Ollie. What shall we do with the one foot? How many feet are there in all? How many yards in six feet? What figure do we write? ***Must*** we write a zero? Why is there not the same need to write a zero here as in simple numbers? What shall we do with the two yards? How many yards are there in all? What figure do we write? Write it, Ollie. How far would the three strings reach? How did you know that you must "carry" for twelve when you added inches, and for three when you added yards? (By the table for Long Measure.)

Here is another string. Measure it, May. How long is it? (4 yds. 2 ft. 6 in.) Write the figures for it, Frank. I will cut a piece off. Measure this piece, May. Can you find how long the other piece is without actually measuring it? How long is the piece, May? (1 yd. 1 ft. 9 in.) You may write the figures for the length *of the piece* beneath the figures for the length of the string before it was cut, and subtract, Frank. What will you subtract first? (9 in.) But there are only 6 in. to take it from. What will you do? How many inches are 1 ft. and 6 in., etc.

NOTE.—Be sure that the children see the resemblance of this work to the work in simple numbers.

FOR STUDY.

1. Harry went to the well for water twice, bringing 3 gallons 2 quarts the first time, and 4 gallons 3 quarts the second time. How much did he bring both times? How much more did he bring the second time than the first?

2. After a merchant cut off 12 yards 2 feet from a piece of ribbon, there remained 8 yards 1 foot. How many yards were in the piece at first?

3. Arthur dug 8 bushels 3 pecks of potatoes, and Tom dug 6 bushels 2 pecks. How many bushels did both dig? How many more did Arthur dig than Tom?

4. Frank weighs 91 pounds 8 ounces, and Walter weighs 87 pounds 10 ounces. How many pounds do both boys together weigh? How much heavier is Frank than Walter?

5. From a barrel containing 41 gallons 2 quarts of oil, a grocer sold 23 gallons 2 quarts 1 pint. How much was left in the barrel?

6. How much must be added to 2 yards 2 feet 10 inches to make 3 yards 2 feet 6 inches?

7. Clarence gathered 2 bushels 3 pecks and 5 quarts of walnuts, and Merton gathered 2 bushels 2 pecks and 7 quarts. How many bushels of nuts did both boys gather? How many more did Clarence gather than Merton?

8. One man bought 2 pounds 6 ounces of steak; another bought 2 pounds 8 ounces; and a third bought 3 pounds 2 ounces. How many pounds did all three buy? How much more did the third man buy than the first?

Copy and add:

(9)

gal.	qt.	pt.
4	2	1
2	1	1
3	3	1

(10)

yd.	ft.	in.
3	2	6
2	1	7
1	2	5

(11)

bu.	pk.	qt.
5	3	6
4	2	7
1	3	3

Copy and subtract:

(12)

bu.	pk.	qt.
7	2	3
2	3	7

(13)

yd.	ft.	in.
5	2	4
4	2	8

(14)

lb.	oz.
6	2
2	9

LESSON 65.

Addition and Subtraction — Concluded.

NOTE.—The purpose of this lesson is to give more practice in adding and subtracting. Some of the problems are more difficult, and many of the numbers are larger. Emphasize the fact that the work is done in the same way, whether the numbers are simple, compound, or mixed.

Give several minutes to rapid oral exercises in addition and subtraction. Drill on the table for addition and subtraction, and on the tables of compound numbers. Let the class work examples with figures at the board. See that they work as rapidly as possible, and train them to prove their results every time.

FOR STUDY.

1. A planing mill sold in one day, 795 feet of pine lumber, 874 feet of poplar, 390 feet of cherry, 576 feet of ash, 437 feet of walnut, and 748 feet of maple. How many feet were sold in all?

2. An army of 5,000 men went into battle. Only 3,672 came out of the battle. How many men were missing?

3. A farmer sold four loads of corn. The first weighed 39.625 bushels, the second 40.125 bushels, the third 38.5 bushels, and the fourth 39.75 bushels. How many bushels did he sell?

4. A man owned a farm containing 159.25 acres. How many acres had he left after selling 39.875 acres?

5. Henry's suit of clothes cost \$$23\frac{5}{8}$, his overcoat \$$18\frac{7}{8}$, his shoes \$$3\frac{4}{8}$, and his hat \$$3\frac{3}{8}$. How many dollars did all cost?

6. A flagstaff $60\frac{5}{12}$ feet high was broken in a storm. The part broken off was $13\frac{7}{12}$ feet long. How many feet remained standing?

7. One man bought 2 pounds 7 ounces of steak, another 1 pound 12 ounces, and another 2 pounds 1 ounce. How many pounds did they all buy?

8. From a roll of ribbon containing 26 yards 1 foot 6 inches, a merchant sold 10 yards 2 feet 9 inches. How many yards were left?

9. A farmer received \$537 for his wheat, \$245 for his corn, and \$165 for his oats. He paid \$236 for rent, \$87 for labor, and \$195 for other expenses. *How many* dollars had he left?

10. From a chest of tea holding 60 pounds, a merchant sold 6 pounds 4 ounces one day and 8 pounds 8 ounces the next day. How many pounds were left?

11. A man who had $4,135 in the bank drew out at one time $237.75, at another time $463.87, and a third time $375.12. He deposited $1,273.65. How many dollars had he in the bank then?

12. Frank rode 32⅜ miles in one day on his bicycle. This was 8¾ miles farther than Walter rode. How many miles did both boys ride?

13. Two men start from two cities 568.09 miles apart, and travel toward each other, one at the rate of 135.7 miles a day and the other at the rate of 216.75 miles a day. How far apart are they at the end of the first day?

14. A farmer raised 78 bushels 2 pecks of potatoes one year, 14 bushels 3 pecks less the next year, and 21 bushels 3 pecks more the third year than the second. How many bushels did he raise in three years? How many more bushels did he raise the third year than the first year?

15. Harry can lift 56½ pounds, which is 8¾ pounds more than Albert can lift. James can lift 12⅝ pounds more than Albert. How many pounds can all three boys lift together? How many more pounds can James lift than Harry?

16. A lady paid $65 for a cloak and $35⅔ less for a dress. How much did she pay for both?

17. A merchant sold flour for $1,595 and gained $165.35. How much would he have received *for it if* he had gained $213.50?

18. A merchant sold goods for $756, which was $126.85 less than they cost. How many dollars would he have gained if he had sold them for $951.50?

Copy and find the sums:

(1)	(2)	(3)	(4)	(5)
$42\frac{3}{4}$	$18\frac{5}{6}$	$32\frac{3}{7}$	$19\frac{3}{8}$	$10\frac{5}{9}$
$13\frac{3}{4}$	$23\frac{5}{6}$	$14\frac{3}{7}$	$62\frac{1}{8}$	$28\frac{1}{9}$
$6\frac{1}{4}$	$7\frac{1}{6}$	$21\frac{6}{7}$	$26\frac{3}{8}$	$31\frac{7}{9}$
$37\frac{3}{4}$	$10\frac{5}{6}$	$16\frac{4}{7}$	$31\frac{5}{8}$	$16\frac{4}{9}$
$2\frac{3}{4}$	$3\frac{1}{6}$	$42\frac{5}{7}$	$17\frac{3}{8}$	$43\frac{2}{9}$

(6)			(7)			(8)		(9)		
bu.	pk.	qt.	yd.	ft.	in.	lb.	oz.	gal.	qt.	pt.
4	2	3	7	1	9	6	12	9	3	1
5	1	4	3	2	6	6	9	2	2	1
3	3	7	5	1	3	9	4	7	1	1
7	1	5	6	1	5	2	7	6	0	1

Copy and find the differences:

(10)	(11)	(12)	(13)	(14)
$49\frac{6}{7}$	$53\frac{3}{8}$	$71\frac{1}{6}$	$13\frac{4}{13}$	$27\frac{1}{8}$
$15\frac{2}{7}$	$24\frac{5}{8}$	$39\frac{5}{6}$	$10\frac{9}{13}$	$16\frac{1}{4}$

(15)	(16)	(17)	(18)	(19)
$21\frac{3}{5}$	$18\frac{1}{2}$	$38\frac{1}{3}$	$50\frac{1}{2}$	$72\frac{7}{9}$
$19\frac{4}{5}$	$10\frac{3}{4}$	$26\frac{1}{6}$	$46\frac{3}{4}$	$65\frac{2}{3}$

(20)		(21)			(22)			(23)			
lb.	oz.	gal.	qt.	pt.	yd.	ft.	in.	bu.	pk.	qt.	pt.
7	9	18	2	1	23	2	5	15	2	5	0
3	13	13	3	1	17	2	9	7	3	5	1

LESSON 66.

Multiplication.

ORAL EXERCISE.

NOTE.—The pupil has not yet had the word *multiply*, but he has had a good deal to do with what it means. The time has now come for him to have the word. He has already learned a good many products; he should now have them all in tabular form as far as "nine times nine." If he finds trouble with any of them, let him form them with his splints, then write the result and commit it to memory. The multiplication table so far as "nine times nine" must be known, or he can not multiply; if he attempts to do so without a knowledge of the table he will only add, or count. As the addition table was a subtraction table as well, so the multiplication table is a division table as well. The pupil made four statements for each combination in the addition table. Train him to make six statements for each combination in the multiplication table as follows: "Four times six are twenty-four," "six times four are twenty-four," "four sixes in twenty-four," "six fours in twenty-four," "one-fourth of twenty-four is six," "one sixth of twenty-four is four." Teach the pupil that the multiplier always means "times," and that the product of each statement is always of the same name or kind as the multiplicand.

How many apples are three times four apples? Six times seven cents? Eight times five? Etc. Such work as you are now doing is called *multiplication*. I will write the word. Notice how it is spelled. (Writing.) The number that we take some number of times is called the *multiplicand*. (Writing it under the other word.) The number which tells how many times we take the multiplicand is called the *multiplier*. (Writing.) The number that we get by multiplying is called the *product*. (Writing.) Four times three apples are how many? What is the multiplicand? What is the multiplier? What is the product? Of what kind is the multiplicand? (Apples.) If we take four times some number of apples, of what kind would you think the product must be? Yes, remember that the product is always the same kind as the multiplicand.

We have learned a good many products, and we will learn some more, perhaps. At any rate, I want you to learn all that you find in the multiplication table, on page 209, at the beginning of the

lesson for study. You will remember that in your addition table you had four ways of saying for each two numbers you added. Can you recall them now? Six and five? Five and six? Eleven less five? Eleven less six? Now, for each two numbers in the multiplication table, I want you to be ready to make six statements. Let us try it. Six times five? Five times six? How many sixes in thirty? How many fives in thirty? One sixth of thirty? One fifth of thirty?

Now, let us do some work in multiplication where we have larger numbers. Thomas, you may take the splints and lay out the numbers I tell you, just as you would if you were going to add them. Lay out four tens and three ones; another four tens and three ones; another four tens and three ones, and another four tens and three ones. How many numbers have you? (Four.) What do you notice about them? (They are all alike.) Mary may write the figures for one of these numbers on the board. How many of these numbers are we going to put together? (Four.) Mary, write the figure 4 under the 3. What figures show what number we are going to take four times? We are going to find how many times forty-three splints? What figure shows that? Then what is the multiplier? Point to it. What is the multiplicand? Point to it. Where is the forty-three that the figures stand for? How many are four times three splints? Pick up the twelve splints, George. What will you do with the twelve splints? (Bunch ten of them.) Lay the two loose splints by themselves; keep the bunch in your hand. Mary, write a figure 2 for the loose splints, just as you did in addition. Four times four bunches are how many bunches? Let George put his bunch with these sixteen bunches. How many bunches are there now? What will you do with the seventeen bunches? (Make a larger bunch with ten of them.) You may do it, Bessie. What is that larger bunch? How many small bunches are left? Lay the seven bunches with the two loose sticks; and Mary may make a 7 in the proper place to stand for them. Now put the large bunch with the rest, and Mary will make a figure 1 for it in the proper place. How many are four times forty-three splints? Where are the splints? Where are the figures that stand for the product? Where are the figures for the multiplicand? Where is the multiplicand itself now? (In the product.)

NOTE.—Have a few minutes' drill on the table.

FOR STUDY.

MULTIPLICATION TABLE.

2 times 1=2
2 times 2=4
2 times 3=6
2 times 4=8
2 times 5=10
2 times 6=12
2 times 7=14
2 times 8=16
2 times 9=18

3 times 3=9
3 times 4=12
3 times 5=15
3 times 6=18
3 times 7=21
3 times 8=24
3 times 9=27

4 times 4=16
4 times 5=20
4 times 6=24
4 times 7=28
4 times 8=32
4 times 9=36

5 times 5=25
5 times 6=30
5 times 7=35
5 times 8=40
5 times 9=45

6 times 6=36
6 times 7=42
6 times 8=48
6 times 9=54

7 times 7=49
7 times 8=56
7 times 9=63
8 times 8=64
8 times 9=72
9 times 9=81

1. What will 8 rugs cost at $7 each? Write no figures.

2. A gardener planted 7 rows of currant bushes, planting 9 in each row. How many currant bushes did he plant? (No figures.)

3. If a quart of peanuts is worth 9 cents, what is a peck worth?

4. How many square inches in a piece of paper 9 inches long and 9 inches wide? Can you explain the example by a diagram?

5. The water in my well is 8½ feet deep, and the well is 8 times as deep. How deep is the well? How many are 8 times ½?

6. If 9 lamps cost $63, what does 1 lamp cost?

7. A man rows 8 miles an hour. How many hours will it take him to row 72 miles?

8. At 84 cents a bushel what will 8 bushels of wheat cost? Write the figures and multiply. Which number is the multiplicand? Which is the multiplier? Which is the product?

9. John's father has 65 sheep and Henry's father has 5 times as many. How many sheep has Henry's father?

10. A man traveled 78 miles by boat and 6 times as far by railroad. How many miles did he travel by railroad? How many miles in all?

11. A farmer put 95 bushels of wheat in each of 4 bins. How many bushels had he?

12. A man earns $36¼ a week. How many dollars will he earn in 8 weeks?

13. How many cubic feet in 9 cubic yards?

14. How many square inches are there in a board 4 feet long and 8 inches wide? Can you explain this as we learned to do long ago?

15. What is the cost of 7 bushels of potatoes at 65 cents a bushel? How many dollars?

16. A merchant bought 8 rolls of cloth, each containing 45 yards. How many yards did he buy?

17. A bushel of corn weighs 56 pounds. How many pounds do 9 bushels weigh?

18. A boy worked 9 days at 85 cents a day. How many dollars did he earn?

19. How many hours are there in one week?

20. How many are:

8 times 96 men?
7 times 78 cows?
5 times 37 feet?
8 times 86 acres?
6 times 76 children?
4 times 87 pounds?
9 times 95 miles?

Write these figures and find the product:

(1)	(2)	(3)	(4)	(5)
46 sheep.	$73	68 books.	87 posts.	79 birds.
6	8	9	3	7
(6)	(7)	(8)	(9)	(10)
34 desks.	54 pens.	76 boys.	98 men.	38 pinks.
9	8	5	6	7

LESSON 67.

Multiplication — Continued.

ORAL EXERCISE.

NOTE.—When the multiplier is expressed by more than one figure, the child should be led to see that the products by the tens, hundreds, etc., of the multiplier are found exactly as the product by the units is found; then these products are multiplied by ten, one hundred, etc., by putting their figures into a new place. For instance, when the multiplier is forty-two, we find two times the multiplicand, then ten times four times the multiplicand, and add these results together. All our actual multiplications are by nine or some smaller number. In multiplying mixed numbers and compound numbers, the pupil should see that he proceeds precisely as in simple numbers, only that, instead of "carrying" invariably for ten, he carries for various numbers.

Practice vigorously on the multiplication table. The nine figures may be written around the circumference of a circle, as in addition; then write multipliers successively in the center, and let the class give products rapidly, as the teacher points to figures around the circle.

All take places at the board. Write four hundred eighty-three for a multiplicand, and twenty-three for a multiplier. We want to find how many times the multiplicand? Find three times the multiplicand, and write the product as you have learned to do. Now find two times the multiplicand and write the figures right below the other product. The first figures show how many times the multiplicand? The second figures show how many times the multiplicand? But we want how many times two times the multiplicand? (Ten times two times the multiplicand.) Yes, because twenty is ten times two. Now, how can we find ten times the second product? (By moving all of its figures one place to the left.) Move them. Now, do you see that the figures would have stood just as they do now, if you had begun to write the product by putting the first figure in tens' place? You may all write your work as I do now. (Writing.)

$$\begin{array}{r l} 483 & \\ \underline{23} & \\ 1449 & = 3 \text{ times } 483. \\ \underline{966\ } & = 10 \text{ times } 2 \text{ times } 483. \\ 11109 & = 23 \text{ times } 483. \end{array}$$

Suppose Jane has three pieces of ribbon, each one yard two feet and seven inches long. How shall we find the length of the ribbon if she lays the three pieces together end to end? How many times the length of one piece will there be? All write the multiplicand as I do. (Writing.)

yd.	ft.	in.
1,	2,	7.

Notice how I use the commas, and how I tell what kind each figure stands for. Now write the multiplier 3 under the 7, and begin to multiply. How many inches are three times seven inches? How many of this kind make one of the next higher? Then how many feet in twenty-one inches, and how many inches are left? Write a 9 in the product for the inches that are left; and what will you do with the one foot? Yes, just as we did with one bunch or ten, in our other work; we carry it to the next product. What is the next product? (Six feet.) How many feet after the one is added? How many yards in seven feet, and how many feet left? Write 1 in the right place for the foot that is left. How many yards will you have in all? Write 5 under the sign for yards. What is our whole product? In the product for inches we carried one for every twelve, and in the product for feet we carried one for every three; but in common or simple numbers we always carry one for every —? (Ten.) Yes, and that is all the difference between multiplying simple numbers and multiplying such numbers as feet and inches, pecks and quarts, and the like. In compound numbers the tables we have learned tell us how many of one kind make one of the next higher.

John, Thomas, and Harry went fishing, and each caught three and three-fourths pounds of fish. How many pounds did all catch? They caught three times how many pounds? Write the multiplicand so:

pounds.
$3\frac{3}{4}$

Now write the multiplier as usual. Three times three fourths are how many fourths? How many fourths of a pound make a pound? What tells you that? (The denominator.) How many pounds to carry? How many fourths left? Write figures for the one fourth. Now, how many pounds have you in all? Then three times three and three fourths pounds are —? (Eleven and one fourth pounds.) And this time you carried one for —? (Four.) Find five times three bushels and three pecks. Notice that you carry in the same way that you carried in the last multiplication.

FOR STUDY.

1. At 28 cents a pound, what will 32 pounds of butter cost? (Make 32 the multiplier. Why?)

2. $16\frac{1}{2}$ feet make one rod. How many feet are there in 18 rods? (Make 18 the multiplier.)

3. If a man sells 18 gallons 3 quarts 1 pint of milk a day, how many gallons will he sell in 6 days?

4. Find the cost of 38 cows at $48 apiece.

5. If I feed my pony $\frac{5}{8}$ of a peck of oats a day, how many pecks do I feed him in a week? (No figures.)

6. A man bought 57 sheep at $4.65 a head. How many dollars did he pay for all of them?

7. James weighs 86 pounds 2 ounces, and his pony weighs 8 times as much. How heavy is his pony?

8. How many square inches are there in 28 square feet?

9. How many square feet are there in a lawn 67 feet long and 38 feet wide? Explain this as we learned to do long ago.

10. A man earns $26⅜ a week. How many dollars will he earn in 8 weeks?

11. What will 26 yards of silk cost at $2.45 a yard?

12. Two boys gathered 9 bushels of nuts, and sold them at $.45 a peck. How many dollars did they get for the nuts?

13. There are 1,728 cubic inches in one cubic foot. How many cubic inches in 5 cubic feet?

14. What will 9 acres of land cost at $86¼ an acre?

15. At 18 cents an inch, what will 3 yards of gold wire cost?

16. How many cubic inches of water will a pan hold that is 15 inches long, 11 inches wide, and 7 inches deep? Explain this as we learned to do before.

17. At $.95 a bushel, what will 68 bushels of wheat cost?

18. If a train moves at the rate of 35 miles an hour, how many miles will it go in 24 hours?

19. How many melons must I have for 28 children if I give each child one fourth of a melon? (No figures.)

20. If Dan rows 3⅝ miles an hour, how far will he row in 8 hours? (No figures.)

21. If a man digs 13 bushels 3 pecks of potatoes a day, how many bushels will he dig in 6 *days?*

22. If a man buys 35 sheep at $4.35 a head, and 28 more at $4.58 a head, how many dollars does he pay for all of them?

23. If a grocer buys 16 bushels of potatoes at $.75 a bushel and sells them for 28 cents a peck, how much does he gain? How many cents does he gain on one bushel?

24. What will 9 barrels of flour cost at $5⅘ a barrel? (No figures.)

25. What will 6⅓ pounds of uncolored Japan tea cost at 6 cents an ounce? (Make 6⅓ the multiplier.)

26. What must I pay for 17 horses at $185 apiece?

Copy and find products:

(1) $86 8	(2) $3.17 9	(3) 278 houses. 7
(4) 845 acres. 6	(5) 387 yards. 8	(6) 724 men. 9
(7) 409 horses. 7	(8) 697 children. 6	(9) 2017 8
(10) 718 7	(11) 208 9	(12) 396 8

LESSON 68.

Multiplication — Concluded.

ORAL EXERCISE.

NOTE.—The object of this lesson is to impress more fully upon the class that all multiplication is essentially of the same kind, and is performed in the same way. In simple numbers we always "carry" for ten; in fractions and compound numbers we carry for other numbers than ten—such numbers as the denominators or the tables give us. Drill the class for a few minutes on the multiplication table, using the figures around the circle, and in other ways. If any pupil is troubled to remember some product, it may be well to let him get his splints or other counters, and make it; then drill him in committing it to memory. Do not forget to impress the truth that the order of the factors is not material; seven times six is the same as six times seven, etc.

Now all may go to the board. Find the product of seven times two hundred forty-seven. For what number did you "carry" every time?

Now find the product of seven times two hundred forty-seven hundredths of a dollar. For what number did you "carry" this time? What was your multiplicand? What kind is your product?

Find the product of seven times four and five sixths dollars. For what did you carry this time? How did you know that you must "carry" one for six?

Find the product of seven times two bushels five pecks and two quarts. For what number did you "carry" when you multiplied quarts? For what number when you multiplied pecks? How did you know that you must carry for eight and for four?

Find the product of seven times eight ninths of an acre. For what number did you "carry" now? How did you know that you must carry for nine?

Once more, find the product of seven times two feet and six inches. For what did you "carry" when you multiplied inches? For what when you multiplied feet? How did you know that you must carry for twelve and three?

You may now learn a new sign; this is the sign for multiplication. (Making × on the board.) It means "multiplied by." Read these expressions as I write them. (Writing, 6×4, 7×9, 8 men × 5, *etc.)*

FOR STUDY.

1. There are 320 rods in a mile. How many rods are there in 25 miles?

2. If a postman in delivering mail takes 135 steps a minute, how many steps will he take in an hour?

3. A merchant bought 36 boxes of soap at $4.25 a box. How many dollars did he pay for the soap?

4. Find the cost of 9 pounds of coffee at 28⅙ cents a pound.

5. What will a quarter of beef weighing 167 pounds cost, at 8 cents a pound? What will it cost at 1 cent a pound? How many times $1.67 at 8 cents a pound?

6. A merchant sold a barrel of coal-oil, holding 52 gallons, at 5 cents a quart. How much did he receive for it?

7. What must I pay for a railroad ticket for 8 miles, at 2¾ cents a mile? (No figures.)

8. A merchant bought 63 gallons of molasses at $.49 a gallon, and sold it at $.15 a quart. How many dollars did he gain? How many cents on one gallon?

9. A farmer sold seven bags of clover-seed, each holding 2 bushels 3 pecks 5 quarts. How many bushels did he sell?

10. A clock that strikes the hours only, strikes 156 times a day. How many times does it strike during the month of January?

11. What will 7 tons of coal cost when coal is selling at $6⅞ a ton?

18. A merchant sold goods for $756, which was $126.85 less than they cost. How many dollars would he have gained if he had sold them for $951.50?

Copy and find the sums:

(1)	(2)	(3)	(4)	(5)
$42\frac{3}{4}$	$18\frac{5}{6}$	$32\frac{3}{7}$	$19\frac{3}{8}$	$10\frac{5}{9}$
$13\frac{2}{4}$	$23\frac{2}{6}$	$14\frac{2}{7}$	$62\frac{4}{8}$	$28\frac{1}{9}$
$6\frac{1}{4}$	$7\frac{1}{6}$	$21\frac{6}{7}$	$26\frac{7}{8}$	$31\frac{7}{9}$
$37\frac{2}{4}$	$10\frac{3}{6}$	$16\frac{4}{7}$	$31\frac{5}{8}$	$16\frac{4}{9}$
$2\frac{3}{4}$	$3\frac{4}{6}$	$42\frac{5}{7}$	$17\frac{3}{8}$	$43\frac{2}{9}$

(6)

bu.	pk.	qt.
4	2	3
5	1	4
3	3	7
7	1	5

(7)

yd.	ft.	in.
7	1	9
3	2	6
5	1	3
6	1	5

(8)

lb.	oz.
6	12
6	9
9	4
2	7

(9)

gal.	qt.	pt.
9	3	1
2	2	1
7	1	1
6	0	1

Copy and find the differences:

(10)	(11)	(12)	(13)	(14)
$49\frac{6}{7}$	$53\frac{3}{8}$	$71\frac{1}{6}$	$13\frac{4}{13}$	$27\frac{1}{8}$
$15\frac{2}{7}$	$24\frac{5}{8}$	$39\frac{5}{6}$	$10\frac{9}{13}$	$16\frac{1}{4}$

(15)	(16)	(17)	(18)	(19)
$21\frac{3}{5}$	$18\frac{1}{2}$	$38\frac{1}{3}$	$50\frac{1}{2}$	$72\frac{7}{9}$
$19\frac{4}{5}$	$10\frac{3}{4}$	$26\frac{1}{6}$	$46\frac{3}{4}$	$65\frac{2}{3}$

(20)

lb.	oz.
7	9
3	*13*

(21)

gal.	qt.	pt.
18	2	1
13	3	1

(22)

yd.	ft.	in.
23	2	5
17	2	9

(23)

bu.	pk.	qt.	pt.
15	2	5	0
7	3	5	1

LESSON 66.

Multiplication.

ORAL EXERCISE.

NOTE.—The pupil has not yet had the word *multiply*, but he has had a good deal to do with what it means. The time has now come for him to have the word. He has already learned a good many products; he should now have them all in tabular form as far as "nine times nine." If he finds trouble with any of them, let him form them with his splints, then write the result and commit it to memory. The multiplication table so far as "nine times nine" must be known, or he can not multiply; if he attempts to do so without a knowledge of the table he will only add, or count. As the addition table was a subtraction table as well, so the multiplication table is a division table as well. The pupil made four statements for each combination in the addition table. Train him to make six statements for each combination in the multiplication table as follows: "Four times six are twenty-four," "six times four are twenty-four," "four sixes in twenty-four," "six fours in twenty-four," "one-fourth of twenty-four is six," "one sixth of twenty-four is four." Teach the pupil that the multiplier always means "times," and that the product of each statement is always of the same name or kind as the multiplicand.

How many apples are three times four apples? Six times seven cents? Eight times five? Etc. Such work as you are now doing is called *multiplication*. I will write the word. Notice how it is spelled. (Writing.) The number that we take some number of times is called the *multiplicand*. (Writing it under the other word.) The number which tells how many times we take the multiplicand is called the *multiplier*. (Writing.) The number that we get by multiplying is called the *product*. (Writing.) Four times three apples are how many? What is the multiplicand? What is the multiplier? What is the product? Of what kind is the multiplicand? (Apples.) If we take four times some number of apples, of what kind would you think the product must be? Yes, remember that the product is always the same kind as the multiplicand.

We have learned a good many products, and we will learn some more, perhaps. At any rate, I want you to learn all that you find in the multiplication table, on page 209, at the beginning of the

18. A merchant sold goods for $756, which was $126.85 less than they cost. How many dollars would he have gained if he had sold them for $951.50?

Copy and find the sums:

(1)	(2)	(3)	(4)	(5)
$42\frac{3}{4}$	$18\frac{5}{6}$	$32\frac{3}{7}$	$19\frac{3}{8}$	$10\frac{5}{9}$
$13\frac{3}{4}$	$23\frac{1}{6}$	$14\frac{2}{7}$	$62\frac{4}{8}$	$28\frac{1}{9}$
$6\frac{1}{4}$	$7\frac{1}{6}$	$21\frac{6}{7}$	$26\frac{7}{8}$	$31\frac{7}{9}$
$37\frac{3}{4}$	$10\frac{5}{6}$	$16\frac{4}{7}$	$31\frac{5}{8}$	$16\frac{4}{9}$
$2\frac{3}{4}$	$3\frac{4}{6}$	$42\frac{5}{7}$	$17\frac{3}{8}$	$43\frac{2}{9}$

(6)

bu.	pk.	qt.
4	2	3
5	1	4
3	3	7
7	1	5

(7)

yd.	ft.	in.
7	1	9
3	2	6
5	1	3
6	1	5

(8)

lb.	oz.
6	12
6	9
9	4
2	7

(9)

gal.	qt.	pt.
9	3	1
2	2	1
7	1	1
6	0	1

Copy and find the differences:

(10)	(11)	(12)	(13)	(14)
$49\frac{6}{7}$	$53\frac{3}{8}$	$71\frac{1}{6}$	$13\frac{4}{13}$	$27\frac{1}{8}$
$15\frac{2}{7}$	$24\frac{5}{8}$	$39\frac{5}{6}$	$10\frac{9}{13}$	$16\frac{1}{4}$

(15)	(16)	(17)	(18)	(19)
$21\frac{3}{5}$	$18\frac{1}{2}$	$38\frac{1}{3}$	$50\frac{1}{2}$	$72\frac{1}{9}$
$19\frac{4}{5}$	$10\frac{3}{4}$	$26\frac{1}{6}$	$46\frac{3}{4}$	$65\frac{2}{3}$

(20)

lb.	oz.
7	9
3	*13*

(21)

gal.	qt.	pt.
18	2	1
13	3	1

(22)

yd.	ft.	in.
23	2	5
17	2	9

(23)

bu.	pk.	qt.	pt.
15	2	5	0
7	3	5	1

LESSON 66.

Multiplication.

ORAL EXERCISE.

NOTE.—The pupil has not yet had the word *multiply*, but he has had a good deal to do with what it means. The time has now come for him to have the word. He has already learned a good many products; he should now have them all in tabular form as far as "nine times nine." If he finds trouble with any of them, let him form them with his splints, then write the result and commit it to memory. The multiplication table so far as "nine times nine" must be known, or he can not multiply; if he attempts to do so without a knowledge of the table he will only add, or count. As the addition table was a subtraction table as well, so the multiplication table is a division table as well. The pupil made four statements for each combination in the addition table. Train him to make six statements for each combination in the multiplication table as follows: "Four times six are twenty-four," "six times four are twenty-four," "four sixes in twenty-four," "six fours in twenty-four," "one-fourth of twenty-four is six," "one sixth of twenty-four is four." Teach the pupil that the multiplier always means "times," and that the product of each statement is always of the same name or kind as the multiplicand.

How many apples are three times four apples? Six times seven cents? Eight times five? Etc. Such work as you are now doing is called *multiplication*. I will write the word. Notice how it is spelled. (Writing.) The number that we take some number of times is called the *multiplicand*. (Writing it under the other word.) The number which tells how many times we take the multiplicand is called the *multiplier*. (Writing.) The number that we get by multiplying is called the *product*. (Writing.) Four times three apples are how many? What is the multiplicand? What is the multiplier? What is the product? Of what kind is the multiplicand? (Apples.) If we take four times some number of apples, of what kind would you think the product must be? Yes, remember that the product is always the same kind as the multiplicand.

We have learned a good many products, and we will learn some more, perhaps. At any rate, I want you to learn all that you find in the multiplication table, on page 209, at the beginning of the

18. A merchant sold goods for $756, which was $126.85 less than they cost. How many dollars would he have gained if he had sold them for $951.50?

Copy and find the sums:

(1)	(2)	(3)	(4)	(5)
$42\frac{3}{4}$	$18\frac{5}{6}$	$32\frac{3}{7}$	$19\frac{3}{8}$	$10\frac{5}{9}$
$13\frac{2}{4}$	$23\frac{2}{6}$	$14\frac{2}{7}$	$62\frac{4}{8}$	$28\frac{1}{9}$
$6\frac{1}{4}$	$7\frac{1}{6}$	$21\frac{6}{7}$	$26\frac{2}{8}$	$31\frac{7}{9}$
$37\frac{2}{4}$	$10\frac{3}{6}$	$16\frac{4}{7}$	$31\frac{5}{8}$	$16\frac{4}{9}$
$2\frac{3}{4}$	$3\frac{4}{6}$	$42\frac{5}{7}$	$17\frac{3}{8}$	$43\frac{2}{9}$

(6)			(7)			(8)		(9)		
bu.	pk.	qt.	yd.	ft.	in.	lb.	oz.	gal.	qt.	pt.
4	2	3	7	1	9	6	12	9	3	1
5	1	4	3	2	6	6	9	2	2	1
3	3	7	5	1	3	9	4	7	1	1
7	1	5	6	1	5	2	7	6	0	1

Copy and find the differences:

(10)	(11)	(12)	(13)	(14)
$49\frac{6}{7}$	$53\frac{3}{8}$	$71\frac{1}{6}$	$13\frac{4}{13}$	$27\frac{1}{8}$
$15\frac{2}{7}$	$24\frac{5}{8}$	$39\frac{5}{6}$	$10\frac{9}{13}$	$16\frac{1}{4}$

(15)	(16)	(17)	(18)	(19)
$21\frac{3}{5}$	$18\frac{1}{2}$	$38\frac{1}{3}$	$50\frac{1}{2}$	$72\frac{7}{9}$
$19\frac{4}{5}$	$10\frac{3}{4}$	$26\frac{1}{6}$	$46\frac{3}{4}$	$65\frac{2}{3}$

(20)		(21)			(22)			(23)			
lb.	oz.	gal.	qt.	pt.	yd.	ft.	in.	bu.	pk.	qt.	pt.
7	9	18	2	1	23	2	5	15	2	5	0
3	13	13	3	1	17	2	9	7	3	5	1

LESSON 66.

Multiplication.

ORAL EXERCISE.

NOTE.—The pupil has not yet had the word *multiply*, but he has had a good deal to do with what it means. The time has now come for him to have the word. He has already learned a good many products; he should now have them all in tabular form as far as "nine times nine." If he finds trouble with any of them, let him form them with his splints, then write the result and commit it to memory. The multiplication table so far as "nine times nine" must be known, or he can not multiply; if he attempts to do so without a knowledge of the table he will only add, or count. As the addition table was a subtraction table as well, so the multiplication table is a division table as well. The pupil made four statements for each combination in the addition table. Train him to make six statements for each combination in the multiplication table as follows: "Four times six are twenty-four," "six times four are twenty-four," "four sixes in twenty-four," "six fours in twenty-four," "one-fourth of twenty-four is six," "one sixth of twenty-four is four." Teach the pupil that the multiplier always means "times," and that the product of each statement is always of the same name or kind as the multiplicand.

How many apples are three times four apples? Six times seven cents? Eight times five? Etc. Such work as you are now doing is called *multiplication*. I will write the word. Notice how it is spelled. (Writing.) The number that we take some number of times is called the *multiplicand*. (Writing it under the other word.) The number which tells how many times we take the multiplicand is called the *multiplier*. (Writing.) The number that we get by multiplying is called the *product*. (Writing.) Four times three apples are how many? What is the multiplicand? What is the multiplier? What is the product? Of what kind is the multiplicand? (Apples.) If we take four times some number of apples, of what kind would you think the product must be? Yes, remember that the product is always the same kind as the multiplicand.

We have learned a good many products, and we will learn some more, perhaps. At any rate, I want you to learn all that you find in the multiplication table, on page 209, at the beginning of the

lesson for study. You will remember that in your addition table you had four ways of saying for each two numbers you added. Can you recall them now? Six and five? Five and six? Eleven less five? Eleven less six? Now, for each two numbers in the multiplication table, I want you to be ready to make six statements. Let us try it. Six times five? Five times six? How many sixes in thirty? How many fives in thirty? One sixth of thirty? One fifth of thirty?

Now, let us do some work in multiplication where we have larger numbers. Thomas, you may take the splints and lay out the numbers I tell you, just as you would if you were going to add them. Lay out four tens and three ones; another four tens and three ones; another four tens and three ones, and another four tens and three ones. How many numbers have you? (Four.) What do you notice about them? (They are all alike.) Mary may write the figures for one of these numbers on the board. How many of these numbers are we going to put together? (Four.) Mary, write the figure 4 under the 3. What figures show what number we are going to take four times? We are going to find how many times forty-three splints? What figure shows that? Then what is the multiplier? Point to it. What is the multiplicand? Point to it. Where is the forty-three that the figures stand for? How many are four times three splints? Pick up the twelve splints, George. What will you do with the twelve splints? (Bunch ten of them.) Lay the two loose splints by themselves; keep the bunch in your hand. Mary, write a figure 2 for the loose splints, just as you did in addition. Four times four bunches are how many bunches? Let George put his bunch with these sixteen bunches. How many bunches are there now? What will you do with the seventeen bunches? (Make a larger bunch with ten of them.) You may do it, Bessie. What is that larger bunch? How many small bunches are left? Lay the seven bunches with the two loose sticks; and Mary may make a 7 in the proper place to stand for them. Now put the large bunch with the rest, and Mary will make a figure 1 for it in the proper place. How many are four times forty-three splints? Where are the splints? Where are the figures that stand for the product? Where are the figures for the multiplicand? Where is the multiplicand itself now? (In the product.)

Note.—Have a few minutes' drill on the table.

FOR STUDY.

MULTIPLICATION TABLE.

2 times 1=2
2 times 2=4
2 times 3=6
2 times 4=8
2 times 5=10
2 times 6=12
2 times 7=14
2 times 8=16
2 times 9=18

3 times 3=9
3 times 4=12
3 times 5=15
3 times 6=18
3 times 7=21
3 times 8=24
3 times 9=27

4 times 4=16
4 times 5=20
4 times 6=24
4 times 7=28
4 times 8=32
4 times 9=36

5 times 5=25
5 times 6=30
5 times 7=35
5 times 8=40
5 times 9=45

6 times 6=36
6 times 7=42
6 times 8=48
6 times 9=54

7 times 7=49
7 times 8=56
7 times 9=63
8 times 8=64
8 times 9=72
9 times 9=81

1. What will 8 rugs cost at $7 each? Write no figures.

2. A gardener planted 7 rows of currant bushes, planting 9 in each row. How many currant bushes did he plant? (No figures.)

3. If a quart of peanuts is worth 9 cents, what is a peck worth?

4. How many square inches in a piece of paper 9 inches long and 9 inches wide? Can you explain the example by a diagram?

5. The water in my well is 8½ feet deep, and the well is 8 times as deep. How deep is the well? How many are 8 times ½?

6. If 9 lamps cost $63, what does 1 lamp cost?

7. A man rows 8 miles an hour. How many hours will it take him to row 72 miles?

8. At 84 cents a bushel what will 8 bushels of wheat cost? Write the figures and multiply. Which number is the multiplicand? Which is the multiplier? Which is the product?

9. John's father has 65 sheep and Henry's father has 5 times as many. How many sheep has Henry's father?

10. A man traveled 78 miles by boat and 6 times as far by railroad. How many miles did he travel by railroad? How many miles in all?

11. A farmer put 95 bushels of wheat in each of 4 bins. How many bushels had he?

12. A man earns $36⅓ a week. How many dollars will he earn in 8 weeks?

13. How many cubic feet in 9 cubic yards?

14. How many square inches are there in a board 4 feet long and 8 inches wide? Can you explain this as we learned to do long ago?

15. What is the cost of 7 bushels of potatoes at 65 cents a bushel? How many dollars?

16. A merchant bought 8 rolls of cloth, each containing 45 yards. How many yards did he buy?

17. A bushel of corn weighs 56 pounds. How many pounds do 9 bushels weigh?

18. A boy worked 9 days at 85 cents a day. How many dollars did he earn?

19. How many hours are there in one week?

20. How many are:

3 times 96 men?
7 times 78 cows?
5 times 37 feet?
8 times 86 acres?
6 times 76 children?
4 times 87 pounds?
9 times 95 miles?

Write these figures and find the product:

(1)	(2)	(3)	(4)	(5)
46 sheep.	$73	68 books.	87 posts.	79 birds.
6	8	9	3	7

(6)	(7)	(8)	(9)	(10)
34 desks.	54 pens.	76 boys.	98 men.	38 pinks.
9	8	5	6	7

LESSON 67.

Multiplication — Continued.

ORAL EXERCISE.

NOTE.—When the multiplier is expressed by more than one figure, the child should be led to see that the products by the tens, hundreds, etc., of the multiplier are found exactly as the product by the units is found; then these products are multiplied by ten, one hundred, etc., by putting their figures into a new place. For instance, when the multiplier is forty-two, we find two times the multiplicand, then ten times four times the multiplicand, and add these results together. All our actual multiplications are by nine or some smaller number. In multiplying mixed numbers and compound numbers, the pupil should see that he proceeds precisely as in simple numbers, only that, instead of "carrying" invariably for ten, he carries for various numbers.

Practice vigorously on the multiplication table. The nine figures may be written around the circumference of a circle, as in addition; then write multipliers successively in the center, and let the class give products rapidly, as the teacher points to figures around the circle.

All take places at the board. Write four hundred eighty-three for a multiplicand, and twenty-three for a multiplier. We want to find how many times the multiplicand? Find three times the multiplicand, and write the product as you have learned to do. Now find two times the multiplicand and write the figures right below the other product. The first figures show how many times the multiplicand? The second figures show how many times the multiplicand? But we want how many times two times the multiplicand? (Ten times two times the multiplicand.) Yes, because twenty is ten times two. Now, how can we find ten times the second product? (By moving all of its figures one place to the left.) Move them. Now, do you see that the figures would have stood just as they do now, if you had begun to write the product by putting the first figure in tens' place? You may all write your work as I do now. (Writing.)

$$\begin{array}{r l} 483 & \\ \underline{23} & \\ 1449 & = 3 \text{ times } 483. \\ \underline{966\ } & =10 \text{ times } 2 \text{ times } 483. \\ 11109 & =23 \text{ times } 483. \end{array}$$

Suppose Jane has three pieces of ribbon, each one yard two feet and seven inches long. How shall we find the length of the ribbon if she lays the three pieces together end to end? How many times the length of one piece will there be? All write the multiplicand as I do. (Writing.)

yd. ft. in.
1, 2, 7.

Notice how I use the commas, and how I tell what kind each figure stands for. Now write the multiplier 3 under the 7, and begin to multiply. How many inches are three times seven inches? How many of this kind make one of the next higher? Then how many feet in twenty-one inches, and how many inches are left? Write a 9 in the product for the inches that are left; and what will you do with the one foot? Yes, just as we did with one bunch or ten, in our other work; we carry it to the next product. What is the next product? (Six feet.) How many feet after the one is added? How many yards in seven feet, and how many feet left? Write 1 in the right place for the foot that is left. How many yards will you have in all? Write 5 under the sign for yards. What is our whole product? In the product for inches we carried one for every twelve, and in the product for feet we carried one for every three; but in common or simple numbers we always carry one for every —? (Ten.) Yes, and that is all the difference between multiplying simple numbers and multiplying such numbers as feet and inches, pecks and quarts, and the like. In compound numbers the tables we have learned tell *us how many* of one kind make one of the next higher.

John, Thomas, and Harry went fishing, and each caught three and three-fourths pounds of fish. How many pounds did all catch? They caught three times how many pounds? Write the multiplicand so:

pounds.
3¾

Now write the multiplier as usual. Three times three fourths are how many fourths? How many fourths of a pound make a pound? What tells you that? (The denominator.) How many pounds to carry? How many fourths left? Write figures for the one fourth. Now, how many pounds have you in all? Then three times three and three fourths pounds are —? (Eleven and one fourth pounds.) And this time you carried one for —? (Four.) Find five times three bushels and three pecks. Notice that you carry in the same way that you carried in the last multiplication.

FOR STUDY.

1. At 28 cents a pound, what will 32 pounds of butter cost? (Make 32 the multiplier. Why?)
2. 16½ feet make one rod. How many feet are there in 18 rods? (Make 18 the multiplier.)
3. If a man sells 18 gallons 3 quarts 1 pint of milk a day, how many gallons will he sell in 6 days?
4. Find the cost of 38 cows at $48 apiece.
5. If I feed my pony ⅝ of a peck of oats a day, how many pecks do I feed him in a week? (No figures.)
6. A man bought 57 sheep at $4.65 a head. How many dollars did he pay for all of them?
7. James weighs 86 pounds 2 ounces, and his pony weighs 8 times as much. How heavy is his pony?
8. How many square inches are there in 28 square feet?

9. How many square feet are there in a lawn 67 feet long and 38 feet wide? Explain this as we learned to do long ago.

10. A man earns $\$26\frac{3}{8}$ a week. How many dollars will he earn in 8 weeks?

11. What will 26 yards of silk cost at $2.45 a yard?

12. Two boys gathered 9 bushels of nuts, and sold them at $.45 a peck. How many dollars did they get for the nuts?

13. There are 1,728 cubic inches in one cubic foot. How many cubic inches in 5 cubic feet?

14. What will 9 acres of land cost at $\$86\frac{3}{4}$ an acre?

15. At 18 cents an inch, what will 3 yards of gold wire cost?

16. How many cubic inches of water will a pan hold that is 15 inches long, 11 inches wide, and 7 inches deep? Explain this as we learned to do before.

17. At $.95 a bushel, what will 68 bushels of wheat cost?

18. If a train moves at the rate of 35 miles an hour, how many miles will it go in 24 hours?

19. How many melons must I have for 28 children if I give each child one fourth of a melon? (No figures.)

20. If Dan rows $3\frac{5}{8}$ miles an hour, how far will he row in 8 hours? (No figures.)

21. If a man digs 13 bushels 3 pecks of potatoes a day, how many bushels will he dig in 6 *days?*

22. If a man buys 35 sheep at $4.35 a head, and 28 more at $4.58 a head, how many dollars does he pay for all of them?

23. If a grocer buys 16 bushels of potatoes at $.75 a bushel and sells them for 28 cents a peck, how much does he gain? How many cents does he gain on one bushel?

24. What will 9 barrels of flour cost at $5⅘ a barrel? (No figures.)

25. What will 6½ pounds of uncolored Japan tea cost at 6 cents an ounce? (Make 6½ the multiplier.)

26. What must I pay for 17 horses at $185 apiece?

Copy and find products:

(1)	(2)	(3)
$86	$3.17	278 houses.
8	9	7

(4)	(5)	(6)
845 acres.	387 yards.	724 men.
6	8	9

(7)	(8)	(9)
409 horses.	697 children.	2017
7	6	8

(10)	(11)	(12)
718	208	396
7	9	8

LESSON 68.

Multiplication — Concluded.

ORAL EXERCISE.

NOTE.—The object of this lesson is to impress more fully upon the class that all multiplication is essentially of the same kind, and is performed in the same way. In simple numbers we always "carry" for ten; in fractions and compound numbers we carry for other numbers than ten—such numbers as the denominators or the tables give us. Drill the class for a few minutes on the multiplication table, using the figures around the circle, and in other ways. If any pupil is troubled to remember some product, it may be well to let him get his splints or other counters, and make it; then drill him in committing it to memory. Do not forget to impress the truth that the order of the factors is not material; seven times six is the same as six times seven, etc.

Now all may go to the board. Find the product of seven times two hundred forty-seven. For what number did you "carry" every time?

Now find the product of seven times two hundred forty-seven hundredths of a dollar. For what number did you "carry" this time? What was your multiplicand? What kind is your product?

Find the product of seven times four and five sixths dollars. For what did you carry this time? How did you know that you must "carry" one for six?

Find the product of seven times two bushels five pecks and two quarts. For what number did you "carry" when you multiplied quarts? For what number when you multiplied pecks? How did you know that you must carry for eight and for four?

Find the product of seven times eight ninths of an acre. For what number did you "carry" now? How did you know that you must carry for nine?

Once more, find the product of seven times two feet and six inches. For what did you "carry" when you multiplied inches? For what when you multiplied feet? How did you know that you must carry for twelve and three?

You may now learn a new sign; this is the sign for multiplication. (Making × on the board.) It means "multiplied by." Read these expressions as I write them. (Writing, 6×4, 7×9, 8 men × 5, *etc.)*

FOR STUDY.

1. There are 320 rods in a mile. How many rods are there in 25 miles?

2. If a postman in delivering mail takes 135 steps a minute, how many steps will he take in an hour?

3. A merchant bought 36 boxes of soap at $4.25 a box. How many dollars did he pay for the soap?

4. Find the cost of 9 pounds of coffee at $28\frac{1}{6}$ cents a pound.

5. What will a quarter of beef weighing 167 pounds cost, at 8 cents a pound? What will it cost at 1 cent a pound? How many times $1.67 at 8 cents a pound?

6. A merchant sold a barrel of coal-oil, holding 52 gallons, at 5 cents a quart. How much did he receive for it?

7. What must I pay for a railroad ticket for 8 miles, at $2\frac{3}{4}$ cents a mile? (No figures.)

8. A merchant bought 63 gallons of molasses at $.49 a gallon, and sold it at $.15 a quart. How many dollars did he gain? How many cents on one gallon?

9. A farmer sold seven bags of clover-seed, each holding 2 bushels 3 pecks 5 quarts. How many bushels did he sell?

10. A clock that strikes the hours only, strikes 156 times a day. How many times does it strike during the month of January?

11. What will 7 tons of coal cost when coal is selling at \$$6\frac{7}{8}$ a ton?

12. A merchant bought a cask of vinegar, holding 16 gallons, at 28 cents a gallon, and sold it at 8 cents a quart. How much did he gain?

13. How many cubic inches are there in a block of wood 18 inches long, 10 inches wide, and 8 inches thick? (Explain this.)

14. A family consumes 4 pounds 2 ounces of flour a day. How many pounds will it consume in a week? (No figures.)

15. What will 8 acres of land cost at $67⅞ an acre?

16. A merchant bought 6 pieces of ribbon, each containing 14 yards 2 feet 8 inches. How many yards in all?

17. A farmer sold 36 barrels of apples at $4.35 a barrel. How much did he receive for them?

Copy and find products:

$$\begin{array}{cccc} (1) & (2) & (3) & (4) \\ 49 & 319 & \$10.27 & 9.58 \text{ feet.} \\ \underline{26} & \underline{18} & \underline{65} & \underline{16} \end{array}$$

$$\begin{array}{ccc} (5) & (6) & (7) \\ 7.09 \text{ bushels.} & 56.7 \text{ gallons.} & 4.375 \text{ pounds.} \\ \underline{29} & \underline{53} & \underline{24} \end{array}$$

$$\begin{array}{ccc|cc|ccc} \multicolumn{3}{c}{(8)} & \multicolumn{2}{c}{(9)} & \multicolumn{3}{c}{(10)} \\ \text{bu.} & \text{pk.} & \text{qt.} & \text{lb.} & \text{oz.} & \text{yd.} & \text{ft.} & \text{in.} \\ 3 & 2 & 6 & 27 & 6 & 37 & 1 & 4 \\ & & 7 & & 3 & & & 9 \end{array}$$

(11) 315×4 (12) $4.17×6 (13) .306×72 (14) 2.96×38

LESSON 69.

Division.

ORAL EXERCISE.

NOTE.—Impress upon pupils that the multiplication table is the division table also. This they will understand readily if they have been trained in the six statements for each pair of numbers, as was suggested. Teach long division before short division; the latter is a contracted or abbreviated form. Keep the pupil to the thought that when he finds how many times one number is in another, the divisor and dividend must be of exactly the same kind; and in this case the quotient always means *times*, and nothing else. When we divide to find one of the equal parts of a number, the quotient must be of the same kind as the dividend. A remainder is always of the same kind as the dividend. When the divisor is quite large, the pupil will find difficulty in determining how many times it will be found in any part of the dividend. He can make only a sort of guess at best. If, however, his product of the divisor by his supposed quotient is greater than the part of the dividend he used, he must take a smaller quotient. If the remainder, after subtracting, is as large as the divisor, he must have a larger quotient. All these points he should be led to see by judicious drill at the board, under the direction of the teacher.

Tell me how many fours in sixteen. How many fives in twenty. How many eights in forty. What is one fifth of twenty-five? One seventh of forty-two? Have we had a good deal of such work before? Yes, but we have not had a name for it. Such work is called *Division*. I will write the word here on the board. (Writing.) How many fives in twenty-seven? How many left? Then, did we really divide twenty-seven or twenty-five? What is one sixth of thirty-nine? How many remainder? What did we really divide?

Now we will take an example in division, and work it with our splints, and do some work with figures at the same time, to stand for, or represent, the other work. Seth may lay out in order, seven bunches of one hundred, three bunches of ten, and five loose splints. This is the number to be divided, and it is called the *Dividend*. Mary may write the figures for the dividend on the board, and I will write the word under the word division. We want to find how many times three splints in this number. Mary, did you write the word "splints" after your dividend?

Write it, and draw a curved line at the left of the dividend, and write "3 splints" to the left of that. What number are we going to divide by? Yes, and we call the three splints the *Divisor*. I will write the word below these others. Thomas, hold up the bunches that the 7 stands for. What are they? If they were seven loose splints, how many times should we have three splints in them? And how many would be left? Thomas may lay one bunch with the tens, and Mary may make a curved line at the right of the dividend, and write 2 at the right of it. Thomas, how many times six splints in the six bunches you have? (One hundred times six splints.) Then how many times two times three splints will there be in these bunches? (One hundred times.) So our 2 must stand in hundreds' place. It will when we get done. Two times three are how many? Mary, write the 6 under the 7, and Thomas may lay away his six bunches; we have found out how many times three splints in them. Now, Mary, draw a line under the 6 and write 1 below for the bunch that is left. Thomas, hold up the one hundred that was left. Take off the band. How many tens have you? Put them with the three tens. How many tens? (Thirteen.) Mary, bring down the 3 beside the 1; 13 will stand for these thirteen tens. How many times three splints in thirteen splints? How many times as many in thirteen tens? Then, write the 4 in tens' place at the right of the 2. Write four times 3 under the 13, and draw a line, Mary. Does this use up all the thirteen bunches? How many are left? Lay the twelve bunches away, we have used them, and below the 12 write 1 for the bunch that is left. Take off the band and put the ten with the five loose ones. Mary, put the figure for the five loose ones beside the 1 that stood for the ten that was left. How many times three splints in fifteen splints? Write the 5 in ones' place at the right of the 4. Write five times three under the 15, and subtract. What is left? Then, how many times three splints in seven hundred thirty-five splints? Where are the figures that show this number? We call 245 the *Quotient;* that is the name of the number we get by dividing. I will write the word under the other names. I want you to remember these names, and remember how to spell them.

When we write out all our work as we have done this time, we call the work Long Division. When the divisor is a large number *we have to do it* this way. But when the divisor is small we can save some of the figures. Let us see how. Mary, erase everything but the dividend and the divisor. Now, draw a line under the dividend. How many times three in seven? Write the 2 under the 7. Two times three are how many? Take six from seven. How many left? Count this as ten in your minds and put the 3 with it. How many threes in thirteen? Write the 4 to the right of the 2. Four times three are how many? Twelve from thirteen leave how many? Think of this one that is left as ten, and put it with the 5. How many threes in fifteen? Write the 5 in the proper place. Is anything left? Then our work is done. Where is the quotient now? Read it. Is it the same as before? Yes, and you see we have done the same work as before, only we did a part of it just in the mind, without any figures. When we do the work in this way, we call it Short Division.

Now, Henry may put on again those bands that we took off the splints, and lay the seven hundred thirty-five splints in order once more, and Lillie may make the figures for them on the board. Now I want to find one third of these splints. Write 3 for the divisor as before, Lillie. We will do this work by short division. Draw the line. Does the three stand for splints now? Then we will not write the word splints with it. James, hold up the seven hundreds. Lay one third of them by themselves; Lillie, make the figure 2 for these two hundred under the 7. Are those two bunches one third of seven bunches? One third of how many? (Six.) Put away the six bunches, James, and take the band off the one that is left. How many tens have you now? What is one third of thirteen tens? James may lay the four tens at the right of the two hundreds, and Lillie may make the figure 4 in the right place. Three times four tens are how many tens? Put away the twelve tens, James, and take the band off the one that is left. How many loose ones now? What is one third of fifteen loose ones? Put the five loose ones at the right of the tens, James; and Lillie may make the figure 5 in the right place. Three times five splints are how many? Then lay away the rest of the fifteen. Point to the figures in the quotient. Read them. What do they stand for? (Splints.) Where are the splints they stand for? Write the word "splints" after the quotient figures. Why should this quotient be of the same kind as the dividend? (Because it is a part of the dividend.)

This new sign (÷) means "divided by." Read what I now write on the board. *(Writing, $8÷$4, 6 men÷3, 18÷6, etc.)*

FOR STUDY.

1. At six cents a spool, how many spools of thread can be bought for 54 cents? How many sixes in fifty-four? (No figures.)

2. A baker sold 32 loaves of bread to 8 customers. How many loaves did each customer buy if each bought the same number? What is one eighth of thirty-two?

3. There are 48 desks in a school-room in 6 rows. How many desks are there in each row? One sixth of forty-eight is how many?

4. If you have 56 cents, how many pounds of raisins at 8 cents a pound can you buy? How many eights in fifty-six?

5. Fannie had 72 cents and spent one eighth of them for needles. How many cents did she pay for the needles? One eighth of seventy-two is how many?

6. Elmer has 63 words to copy. How many minutes will it take him to copy them if he copies 7 words a minute? How many sevens in sixty-three?

7. If 8 pints of cream cost 40 cents, how many cents does one pint cost? One eighth of forty is how many?

8. Henry's new book contains 81 pages. How many days will it take him to read it if he reads nine pages a day? How many nines in eighty-one?

9. If 9 sheep cost $45, what was the cost per head? What is one ninth of forty-five?

10. In an orchard are 72 trees. There are 9 trees in a row. How many rows are there?

11. The glass over the picture hanging above my desk contains 63 square inches and is 9 inches long. How wide is it? Can you explain this?

12. At $3 apiece how many hats can be bought for $741? Write figures and divide by short division. Which number is the dividend? Which is the divisor? What is the quotient?

13. A man paid $144 for 3 cows. What was the average price of the cows?

14. If 4 bushels of wheat make a barrel of flour, how many barrels will 376 bushels make?

15. A man paid $660 for 4 horses. What was the average price per head?

16. How many gallons are there in 908 quarts? How many eights in nine hundred eight? How many are left?

17. If the salaries of 5 men are $375 for a month, what is the average salary for each man?

18. A boy sold newspapers at 5 cents apiece, and in the evening he had 485 cents. How many papers had he sold?

19. At 6 miles an hour, how many hours will it take a horse to go 288 miles?

20. A dealer bought 6 pianos, paying $2,790 for them. What was the cost of one piano?

21. A farmer sold 7 bushels of potatoes for 476 cents. What did he receive per bushel for them?

22. How many weeks are there in 973 days?

23. If it takes 8 ounces of tin to make one fruit can, how many cans can be made from 1,264 ounces of tin? How many eights in one thousand two hundred sixty-four?

24. A boy averages 7 miles an hour on his bicycle. In how many hours will he ride 315 miles?

25. How many pecks are there in 1,728 quarts? How many bushels?

26. A farmer raised 1,408 bushels of potatoes on 8 acres. What was the average yield per acre?

27. If 9 pounds of tea cost $7.65, what is the price per pound? What is one ninth of seven dollars and sixty-five cents?

28. How many square yards are there in 1,962 square feet?

29. How many lines of 9 words each in 5,337 words?

Copy and find quotients:

(1)	(2)	(3)	(4)	(5)
3)426	4)788	5)675	6)828	7)1491
(6)	(7)	(8)	(9)	(10)
8)2128	9)567	11)627	12)1728	13)1391

LESSON 70.

Division — Continued.

ORAL EXERCISE.

NOTE.—Give rapid oral exercises combining multiplication and division with addition and subtraction. For instance, say: "Take six, add four, divide by two, multiply by seven, subtract three, divide by four," etc. Give the numbers as fast as the pupils can follow, and let the falling of your voice be the signal for a simultaneous answer. Keep up the drill on the multiplication and division table.

Let the class work several problems at the board together. *Keep all the* work moving as rapidly as possible.

FOR STUDY.

1. If three dozen oranges cost $\$\frac{6}{7}$, what does one dozen cost? What is one third of $\frac{6}{7}$? (No figures.)

2. If 6 horses eat $\frac{18}{20}$ of a ton of hay in a month, what part of a ton does one horse eat, if they eat equal amounts? What is one sixth of $\frac{18}{20}$?

3. Hattie put $\frac{15}{16}$ of a gallon of jelly into 5 glasses of equal size. What part of a gallon does each glass hold? What is one fifth of $\frac{15}{16}$?

4. Hugh sawed $\frac{56}{64}$ of a cord of wood in 7 hours. At that rate what part of a cord did he saw in 1 hour? What is one seventh of $\frac{56}{64}$? What does $\frac{8}{64}$ equal? What is one eighth of $\frac{56}{64}$?

5. A freight train going up the grade near my home went $\frac{72}{80}$ of a mile in 8 minutes. What part of a mile did it average per minute? What is one eighth of $\frac{72}{80}$? What is one ninth of $\frac{72}{80}$?

6. Nine men together weighed $\frac{81}{100}$ of a ton. What part of a ton was the average weight of each man? What is one ninth of $\frac{81}{100}$?

7. James earned $\$7\frac{1}{5}$ in 6 days. How much did he earn a day? How many is one sixth of $7\frac{1}{5}$? (No figures.)

8. At $4 a yard how many yards of silk can be bought for $\$21\frac{4}{8}$? How many times 4 in $21\frac{4}{8}$? (Write the figures and divide.)

9. If a man walks $18\frac{2}{3}$ miles in 4 hours, how many miles does he walk in one hour? What is one fourth of $18\frac{2}{3}$? (No figures.)

10. How many times can a vessel holding 5 gallons be filled from a barrel of oil holding $32\frac{1}{2}$ gallons? How many times 5 in $32\frac{1}{2}$?

11. If 8 cords of wood cost $54⅘, what is the price per cord? What is one eighth of 54⅘?

12. A butcher received $20.56 for beef which he sold at 8 cents a pound. How many pounds did he sell? How many times 8 cents in $20.56?

13. A man earns $87.90 a month, which is 6 times as much as his son earns. How many dollars does his son earn? What is one sixth of $87.90?

14. A farmer put 619 bushels 2 pecks of wheat into 7 bins of equal size. How many bushels did each bin hold? What is one seventh of 619 bushels 2 pecks?

15. At 9 cents a quart how many quarts of berries will cost $13.32? How many times 9 cents in $13.32?

16. A farmer sold 4 cows at the same price per head, receiving $186⅔. What was the price per head? What is one fourth of $186⅔?

17. Andrew caught 7 fish, all together weighing 17 pounds 15 ounces. What was the average weight of each fish? What is one seventh of 17 pounds 15 ounces?

18. Nine girls bought a roll of ribbon containing 58 yards 1 foot 6 inches, which they shared equally. How many yards did each girl receive? What is one ninth of 58 yards 1 foot 6 inches?

19. A lady bought 9 chairs at the same price each, paying $132¾ for all of them. What was the price for each chair? What is one ninth of *$132¾?*

Copy and find the quotients:

(1) $6\overline{)\$559.44}$

(2) $7\overline{)\$269.15}$

(3) $8\overline{)\$542.64}$

(4) $9\overline{)\$760.77}$

(5) $6\overline{)\$568\frac{2}{7}}$

(6) $7\overline{)\$606\frac{2}{3}}$

(7) $8\overline{)\$596\frac{4}{9}}$

(8) $9\overline{)\$610\frac{3}{6}}$

(9)

	yd.	ft.	in.
7)	62	1	3

(10)

	gal.	qt.	pt.
9)	682	3	1

(11)

	bu.	pk.	qt.
6)	230	3	2

(12)

	lbs.	oz.
8)	222	8

LESSON 71.

Division — Concluded.

NOTE.—Keep up the drill in oral combinations. Drill a good deal in finding quotients and remainders, as: How many sixes in thirty-nine? One sixth of thirty-nine? How many eights in fifty-eight? etc. Give rapid exercises in dividing small mixed numbers, as: One third of $7\frac{3}{7}$? One fourth of $5\frac{7}{8}$? etc.

Show that when we find how many times one number in another, the numbers must be of the same kind, as cents in cents, tenths in tenths, sevenths in sevenths, etc.

Keep up the drills on the tables.

Let all the class work examples at the board as rapidly as possible.

FOR STUDY.

1. There are 12 eggs in a dozen. How many dozens are there in 1,476 eggs? (Write figures and divide.)

2. If 14 cows cost $532, what is the cost of one cow?

3. There are 16 ounces in a pound. How many pounds are there in 2,176 ounces? How many times *16 in 2,176*?

4. A farmer sold 16 bushels of potatoes for $10.40. What was the price per bushel? What is one sixteenth of $10.40?

5. A man sold 17 horses at the same price per head, receiving $2,805 for them. What was the price per head?

6. George earns $19 a month. How many months will it take him to earn $684? How many years?

7. A grocer bought butter at 23 cents a pound, paying $11.04. How many pounds did he buy?

8. A man sold 26 acres of land for $1,924. What was the price per acre?

9. A farmer raised 2,380 bushels of potatoes on 28 acres of land. What was the average yield per acre?

10. A fruit grower has 3,072 trees in 32 rows. How many trees are there in one row?

11. How many cubic yards are there in 2,943 cubic feet?

12. There are 36 inches in a yard. How many yards are there in 2,376 inches?

13. A drover bought 45 sheep, paying $218.25. What was the average price per head?

14. There are 52 weeks in a year. How many years are there in 3,536 weeks?

15. A bushel of corn weighs 56 pounds. How many bushels are there in 5,320 pounds?

16. If a vessel sails at the average rate of 63 miles a day, in how many days will it sail 1,701 miles?

17. How many hours are there in 5,820 minutes?

18. A man paid $6,156 for a piece of farming land. He gave $76 an acre. How many acres did he buy?

19. There are 80 chains in a mile. How many miles are there in 9,200 chains? How many rods in a chain?

20. How many companies, averaging 98 men to a company, can be formed from 8,134 men?

21. There are 144 square inches in one square foot. How many square feet are there in 5,328 square inches?

22. One barrel of flour weighs 196 pounds. How many barrels can be filled from 9,408 pounds of flour?

23. There are 231 cubic inches in a gallon of water. How many gallons are there in 8,316 cubic inches of water?

24. There are 365 days in a common year. How many years are there in 5,110 days?

25. A miller sold to a wholesale dealer 218 barrels of flour for $1,962. What was the price paid per barrel by the dealer?

Copy and find quotients:

(1) 21)5187(	(2) 37)8658(	(3) 46)966(
(4) 68)2516(	(5) 85)7905(	(6) 126)4788(
(7) 216)6264(	(8) 324)9396(	(9) 397)5955(
(10) 423)11.421(	(11) 456)20.976(	(12) 583)22.154(

LESSON 72.

Multiplication and Division.

ORAL EXERCISE.

NOTE.—Children should be shown that division is the opposite of multiplication, just as subtraction is the opposite of addition. In multiplication, the factors are put together to make the product. A product is destroyed in division, by taking out one of the factors and leaving the other. In dealing with decimals, it follows, therefore, that if in multiplication the product must have as many decimal places as both the factors, so in division the dividend must have just as many decimal places as both divisor and quotient. On account of the relation of multiplication and division to each other, either can be used to prove the work done by the other.

All go to the board. I want you to find the product of seventy-five tenths multiplied by fourteen tenths. What is meant by multiplying a number by fourteen tenths? (That it is taking fourteen tenths of that number.) Very well; how can we find one tenth of seventy-five tenths? (By moving the figures one place to the right.) Do it. Now you have one tenth of seventy-five tenths. What is the number? What kind? Now, how many times seventy-five hundredths do we want? Find fourteen times seventy-five hundredths. How many have you? What kind? How many decimal places were there in the multiplicand? In the multiplier? How many in the product? Suppose the multiplicand had been hundredths; then how many decimal places would there have been in each? Suppose the multiplier had been hundredths; then how many in each? Do you see, then, that there will always be just as many decimal places in the product as there are in the multiplicand and multiplier counted together? Remember this. Now multiply twelve tenths by three hundredths. What will you find first? When you find one hundredth of twelve tenths (doing the work on the board), how many decimal places must you have? But we have only two figures; what shall we do? (Fill the space between 1 and the point with a zero.)

Four times three are how many? Three times four are how many? What have you been doing? (Multiplying.) Four in twelve how many times? One third of twelve how many? What have you been doing now? (Dividing.) Do you see, then, that dividing is just the opposite of multiplying? By multiplying we twist two numbers together to make another. By dividing we take one number out of another to find what number was twisted with the one we take out to make the dividend. So the dividend in division is the product in multiplication. When we have decimals you have learned that the product must have just as many decimal places as both the other numbers have. Now, don't you see that in division the dividend must have just as many decimal places as the other two numbers have? Remember this.

Because division is the opposite of multiplication we shall see that we can prove our work in one by the other. If we divide the product by the multiplicand, we ought to have—? (The multiplier.) If we divide the product by the multiplier, we ought to have—? (The multiplicand.) If we multiply together the divisor and the quotient, we ought to have—? (The dividend.) All right; remember these answers and prove your work in this way.

Now, all may go to the board and we will work some problems, and see if we can prove our work. We will work some problems in decimals, and see if we can point off our answers correctly.

FOR STUDY.

1. What is the cost of 58 barrels of flour at $3.95 a barrel? Of 9.5 barrels? What does 9.5 times $3.95 mean? Prove your work.

2. How many yards of cloth at $1.5 a yard can be bought for $37.5? For $18.75? For $5.625? Prove.

3. There are 31.5 gallons in a barrel. How many gallons are there in 15 barrels? In 9.7 barrels? In 3.65 barrels? In 1.216 barrels? Prove.

4. There are 16.5 feet in a rod. How many rods are there in 445.5 feet? In 136.95 feet?

5. What will be the cost of 8.75 bushels of oats at $.46 a bushel? Cost of 16.5 bushels at $.45?

6. How many bushels of apples at $1.25 can be bought *for* $8.75? For $23.75?

7. If a vessel sails 13.25 miles an hour, how many miles will it sail in 26 hours? In 9.3 hours? In 2.15 hours?

8. In how many hours will it sail 198.75 miles? 22.525 miles?

9. If a rolling mill makes 95.2 tons of iron in 8 hours, how many tons will it make in 1 hour? How many tons will it make in 12 hours? In 5.16 hours?

10. A perch of stone equals 24.75 cubic feet. How many cubic feet are there in 23 perches? In 6.7 perches? In 9.24 perches?

11. How many perches are there in 767.25 cubic feet? In 61.875 cubic feet? In 22.275 cubic feet? Prove all your answers.

Prove your work for the following problems:

Multiply.

1. 1.9 by 17.
2. .58 by 3.6.
3. .135 by 2.8.
4. 47 by .38.
5. 7.2×.08.
6. 98×.007.
7. .06×.95.

Divide.

8. 28.8 by 3.6.
9. 16.53 by .57.
10. 1.794 by .39.
11. .693 by 7.7.
12. 28.35÷63.
13. .247÷.013.
14. 12÷.48.

Promiscuous Problems.

NOTE.—The solution of a concrete problem involves two very different processes. The first is a study of the conditions of the problem and of the relations of the numbers to one another in order to determine what arithmetical operations are to be performed, and how the numbers are to be used in these operations. The second step is the actual performing of the arithmetical operations. The second step only is strictly arithmetical; the first is the use of the reasoning powers to determine what arithmetical *work to do.* The teacher should see to it that the pupil takes no arithmetical step till he sees clearly what step is to be taken; he should allow no working of the problem "to get the answer," this to be followed by an attempt "to explain" the work. Let the explanation go before the work. It is well sometimes to require the pupil to go through the whole problem step by step, telling what is to be done, and why it is to be done, before a single figure is written or a single operation is performed. If the given numbers are so large as to render the analysis difficult, construct another problem with small numbers, exactly like the one in hand; and when the pupil sees clearly how this is solved, then let him return to the original problem. When the arithmetical operations are reached, insist on rapid work; permit no dawdling; encourage the use of the fewest figures possible. Train the pupil to work without figures whenever the numbers are so small that he can do it. Let him learn from the first that figures are only *helps* at any time; they are not *necessary* to arithmetic.

For practice preparatory to this lesson, let several problems be analyzed in the class and the processes be indicated without performing the operations at all. As far as possible let pupils do all the work; the teacher's business is merely to direct and perhaps give a helpful hint from time to time.

FOR STUDY.

1. What will a peck of peanuts cost at 7 cents a quart?

2. If a train goes $\frac{2}{3}$ of a mile a minute, how many miles will it go in 6 minutes? In $8\frac{1}{2}$ minutes?

3. A butcher received \$.72 for beef, which he sold at \$.09 a pound. How many pounds did he sell?

4. If a peck of berries cost \$.64, how much is that a quart?

5. John's slate is 7 inches wide and contains 63 square inches. How long is it?

6. A man earns \$72 a month and his son earns $\frac{5}{8}$ as much. How many dollars does the son earn?

7. A family buys 2 quarts 1 pint of milk a day. How many quarts does it buy in 6 days?

8. If 4 brooms are worth $\$\frac{8}{10}$, what is one broom worth?

9. If a man walks $18\frac{1}{3}$ miles in 5 hours, at what rate does he walk per hour?

10. If a family uses $\frac{3}{16}$ of a pound of butter in one day, how many days will $\frac{12}{16}$ of a pound last?

11. If one pound of coffee cost $\$\frac{3}{10}$, how many pounds can be bought for $\$\frac{9}{10}$?

12. How many spools of thread at $.06 can be bought for $.54?

13. George gave 6 cents to a beggar, which was $\frac{1}{8}$ of his money. How many cents had he at first?

14. What is the cost of $2\frac{2}{5}$ pounds of dates at 10 cents a pound?

15. If a man earns $81 a month and saves $\frac{1}{9}$ of it, in how many months will he save $72?

16. A farmer sold 64 barrels of apples and $\frac{5}{8}$ as many barrels of potatoes. How many barrels of potatoes did he sell?

17. Ella lost $\frac{1}{4}$ of her money, gave $\frac{1}{4}$ of it for a paper of needles, and then had 18 cents left. How many cents had she at first?

18. Find the cost of 9 pounds of nails at $2\frac{2}{3}$ cents a pound.

19. My lot contains $\frac{4}{5}$ of an acre. What part of an acre is $\frac{1}{2}$ of it?

20. If a pound of tea is sold for $\$\frac{1}{2}$, what must I pay for $\frac{1}{2}$ a pound?

21. John had $\$\frac{1}{4}$, and paid $\frac{1}{2}$ of it for a ball. *What part* of a dollar did the ball cost?

22. What is the difference between two surfaces, one 4 feet square and the other containing 20 square feet?

23. If a coat requires $\frac{3}{4}$ of a dozen buttons, how many coats will require $3\frac{3}{4}$ dozens?

24. A man paid $54 for a cow, which was 6 times as much as he paid for her calf. How much did he pay for her calf?

25. A man asked $72 for a horse; but, finding no purchaser at that price, sold it for $\frac{7}{8}$ of the sum. How much did the horse sell for?

26. If $\frac{3}{4}$ of a bushel of nuts fill 4 bags of equal size, what part of a bushel does each bag hold?

27. When 9 cords of wood cost $\$56\frac{1}{4}$, what will 5 cords cost? What will 1 cord cost?

28. How many cubic inches in a 3-inch cube?

29. A boy paid $28 for a suit of clothes, which was $\frac{7}{9}$ of his money. How much money had he?

30. A girl paid $\frac{1}{2}$ of her money for a hat and $\frac{1}{3}$ of it for shoes; she then had $2 left. How many dollars had she at first?

31. If a train moves $\frac{2}{3}$ of a mile a minute, how many minutes will it be going $3\frac{1}{3}$ miles?

32. Among how many children can $\frac{1}{2}$ of a pound of raisins be divided, if each received $\frac{1}{8}$ of a pound?

33. How many penknives at $\$\frac{3}{5}$ apiece will $3 buy?

34. Albert has $\frac{2}{3}$ of a melon. To how many boys can he give $\frac{1}{6}$ of a melon?

35. How many pounds of beef at $\$\frac{1}{8}$ a pound will $\$\frac{3}{4}$ buy?

36. Willie sold his knife for 36 cents and received in payment 3 quarts of chestnuts at 8 cents a quart and the remainder in money. How much money did he get?

37. If a man can chop 16 cords of wood in 8 days, how many cords can he chop in 9 days?

38. How many badges, each $\frac{1}{16}$ of a yard long, can be made from $\frac{7}{8}$ of a yard of ribbon?

39. If 4 men can mow a field in 6 days, how many days will it take 3 men to mow it? In how many days can one man mow it?

40. If 4 men can dig a certain ditch in 6 days, how many men will be required to dig it in 8 days?

WRITTEN EXERCISES.

1. George Washington was born in 1732 and died in 1799. How old was he?

2. Three men bought a horse. The first paid $47, the second $54, and the third as many dollars as the other two together. How many dollars were paid for the horse?

3. How many ounces are there in 36 pounds avoirdupois?

4. How many sheep at $8 apiece can be bought for $2,192?

5. At 25 cents a peck, what will $4\frac{1}{2}$ bushels of apples cost? Can you do this without figures?

6. A man has $96.75. How many more dollars must he have to buy a horse worth $180?

7. If a man husks $449\frac{2}{3}$ bushels of corn in 6 *days, how many* bushels does he average per day?

8. If a merchant buys wagons at \$56 apiece and sells them at \$75 apiece, how many dollars will he gain on 126 wagons? How many dollars will he gain on each?

9. Two men are 235 miles apart. If they travel toward each other, one at the rate of 39 miles a day and the other at the rate of 27 miles a day, how far will they be apart at the end of the second day?

10. If a horse costs \$$176\frac{4}{5}$, what will 9 horses cost at the same price?

11. How many cubic feet are there in 28 cubic yards?

12. A lady spent \$7.50 for a hat, \$23 for a dress, \$18.45 for a cloak, \$2.25 for gloves, and had \$15.65 left. How many dollars had she at first?

13. If a platform requires $10\frac{1}{3}$ yards of carpet, what will the carpet for it cost at \$$\frac{9}{10}$ a yard?

14. A dealer sold coal for \$3,856.15, and gained \$693.97. How much did the coal cost him?

15. Albert weighs 91 pounds 13 ounces; Roy, 89 pounds 11 ounces; George, 85 pounds 15 ounces; and Merton, 83 pounds 10 ounces. How much do they all weigh? How much heavier is Albert than George?

16. A man worked 38 days at \$2.75 a day. How many dollars did he earn?

17. A merchant bought 54 bushels of wheat at $52\frac{1}{2}$ cents a bushel. He sold $\frac{1}{2}$ of it at $56\frac{1}{3}$ cents a bushel, one third of it at $55\frac{5}{6}$ cents a bushel, and the remainder at $57\frac{1}{8}$ cents a bushel. How much did he gain?

18. How many bushels are there in 288 quarts?

19. What is one sixth of 17 weeks 4 days 16 hours 50 minutes 30 seconds?

20. At 5 cents a quart, what will 45 gallons of milk cost?

21. If it requires 1,218 bricks to build 58 cubic feet of wall, how many bricks will it require to build 98 cubic feet? How many to build one foot?

22. If a pulp mill uses $3\frac{1}{5}$ cords of spruce in 4 hours, how many cords will it use in 6 days of 8 hours each?

23. The top of our dining-table contains 2 square yards, 8 square feet, 126 square inches. How many square yards, square feet, and square inches are there in the top of 8 such tables?

24. A man's carriage cost $165 and his horses 3 times as much. What was the cost of the horses and carriage? How many times $165?

25. John found after paying $\$18\frac{2}{5}$ for a suit of clothes, that he had $\$31\frac{6}{10}$ left. How many dollars had he at first?

26. A farmer took 27 pounds of butter and 38 dozens of eggs to a store. How much did he receive for them if butter was worth $.24 a pound and eggs $.18 per dozen?

27. A man sold his farm of 80 acres at $\$93\frac{1}{2}$ an acre, and with the proceeds bought another farm at $68 an acre. How many acres are there in the latter farm?

28. How many square inches are there in 3 *square yards?*

29. How many yards of carpet, at $1.65 a yard, can be bought for $62.70?

30. How many square feet are there in a floor 18 feet long and 14 feet wide?

31. A boy sold 6 bushels 2 pecks of chestnuts at 15 cents a quart. How many dollars did he receive for them?

32. A coal dealer sold 4 loads of coal weighing respectively 2.25 tons, 1.875 tons, 3.1 tons, and 2.625 tons. How many tons were there in the 4 loads?

33. A clerk's salary is $600 a year. If she uses $48 to purchase books and $296 for board and other expenses, how much will she save in 8 years?

34. At 3 cents a yard, what will 2,700 inches of wire cost?

35. A miller manufactured 650 barrels of flour which he sold at $9 a barrel. The wheat cost $2,860 and the expense of running the mill was $915. How many dollars did he gain on the sale?

36. A bushel of corn weighs 56 pounds. How many bushels are there in a load weighing 2,161.6 pounds?

37. A man sold a piece of land for $1,320, and gained $486.37. How much did he pay for it?

38. A brick is 8 inches long, 4 inches wide, and 2 inches thick. How many cubic inches are there in it?

39. At $.09 a quart, how many quarts of berries will cost $7.56?

40. How many weeks are there in 9,912 hours?

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41. Walter can ride 38.64 miles a day on his bicycle, and Hugh 49.78 miles. If they start from the same place at the same time and travel in opposite directions, how many days will it take them to be 1,591.56 miles apart?

42. A boy worked 18 weeks at $3.75 a week. He spent $16.35 for a suit of clothes, $1.90 for a hat, $2.75 for a pair of shoes, and $3.15 on an excursion. How much money had he left?

43. In a certain village there are 934 men, 259 more women than men, and as many children as there are men and women together. What is the population of the village?

44. A man sold 48 bushels of potatoes for $36.48 What were 20 bushels worth at the same rate?

45. A bicyclist traveled $51\frac{5}{8}$ miles one day, $45\frac{3}{4}$ miles the next day, and $49\frac{1}{2}$ miles the third day. How many miles did he travel in three days?

46. After a merchant had sold from a roll of ribbon, 8 yards 2 feet 9 inches to one lady, and 5 yards 2 feet 8 inches to another, there were 10 yards 7 inches left. How many yards were there at first?

47. A grocer sold 36 barrels of apples at $3.95 a barrel, and gained $28.80. What did they cost him a barrel?

48. How many square inches are there in 6 square yards, 7 square feet, 77 square inches?

49. At 3 cents a square foot, what will 60 pine boards cost, each board being 16 feet long and 1 *foot wide?*

50. A man paid $660 for a span of horses, which was 4 times as much as he paid for his carriage. How much did both the carriage and horses cost?

51. A man has $$68\frac{3}{16}$. How many more dollars must he have to pay for a surrey which will cost him $175?

52. How many cubic inches in a block of wood 1 foot long, 9 inches wide, and 6 inches thick?

53. A grocer bought 15 barrels of flour at $$8\frac{3}{4}$ a barrel, and sold them at $$9\frac{1}{8}$ a barrel. How many dollars did he gain? How much did he gain on one barrel? On 15 barrels?

54. A man saves $9 a week. How many weeks will it take him to pay for his home, which cost him $1,872? How many years (52 weeks in a year)?

55. A grocer exchanged 450 pounds of sugar, at 6 cents a pound, for eggs at 18 cents a dozen. How many dozens did he get? How many pounds of sugar would a dozen eggs buy?

56. What is the cost of $9\frac{1}{2}$ pounds of tea at 6 cents an ounce?

57. $752 is $186.42 greater than what other number?

58. A gardener has 8,695 cabbage plants in 37 rows. If they are distributed equally, how many plants are there in each row?

59. One side of a square is $\frac{3}{4}$ of a yard. What is the length of the boundary of the square?

60. A man divided 14 gallons, 2 quarts, 1 pint of milk equally among 9 poor families. How much milk did each family get?

61. What will 36 bushels of oats cost at 37½ cents a bushel?

62. A grocer bought 9 barrels of sugar, each weighing 275 pounds, at 4 cents a pound. How much did the sugar cost him?

63. How many pounds of wool at 28 cents a pound are worth 98 yards of cloth at $1.30 a yard?

64. Find the difference between the distance around the school-yard and the distance around the school-house.

65. A farmer received $315.18 for his wool, $157.45 more for his wheat than for his wool, and $184.38 more for his corn than for his wool and wheat together. How many dollars did he receive for all? How many acres of land, at $32 an acre, could he buy with the money he received?

66. A car-line is 12 miles long. If a car makes 6 round trips daily, how many miles will it run during the month of July?

67. A boy husks 3 bushels 3 pecks of corn an hour; how many bushels will he husk in 8 hours?

68. A farmer wished to get $200. He sold 50 barrels of apples at $2.75 a barrel, and enough corn at $.5 a bushel to make up the remainder. How many bushels of corn did he sell?

69. Two men start from the same place and walk in opposite directions, one at the rate of 3¾ miles an hour, and the other at the rate of 3⅞ miles an hour. How far apart will they be at the end of 9 hours?

70. How many cubic yards are there in 476.01 *cubic feet?*

71. A man bought a piece of land at $36 an acre, paying $2,844 for it; he sold it at $44 an acre. How much did he gain? How many dollars did he gain on an acre?

72. A man sold wool for $4,376 and gained $836.45. How much would he have received if he had lost $274.85?

73. At 15 cents a quart, what will 63 gallons 2 quarts of molasses cost?

74. A steamship was 5 days 20 hours 45 minutes sailing between two ports. It was 7 days 9 hours 15 minutes making the return trip. How much longer was it returning than in going?

75. A drover bought 46 cows at $32 a head, and 87 sheep at $8.15 a head. He sold all for $2,500. How many dollars did he gain?

76. A farmer sold his wheat for $247.68, receiving $.96 a bushel. How many bushels did he sell?

77. A man bought 14 tons of hay at $12½ a ton, and sold it at $16⅝ a ton. How many dollars did he gain? How many dollars on one ton?

78. How many pounds of sugar at $.06 a pound can be exchanged for 20 dozens of eggs at $.15 a dozen and 12 pounds of butter at $.24 a pound?

79. A boy has a string 6 yards 1 foot 3 inches long. How many feet and inches must he add to it to make it 15 yards long?

80. The divisor is 5.7 and the dividend is 4.731. What is the quotient?

81. A man bought 87 barrels of apples at $2.35 a barrel. After 13 barrels spoiled, he sold the rest at $2.75 a barrel. How much did he lose?

82. How many minutes are there in a week?

83. A dealer paid $149⅓ for 8 plows. For how much apiece must he sell them to gain $8⅔ on each plow?

84. A man divided 5 bushels of apples equally among 8 boys. What part of a bushel did each boy receive?

85. How many cubic inches of water will a pan hold that is 14 inches long, 8 inches wide, and 5 inches deep? How many gallons? (231 cubic inches make a gallon.)

86. A speculator bought 472 bushels of potatoes at $.68 a bushel. He sold ⅝ of them at $.74 a bushel, and the remainder at $.75 a bushel. How much did he gain?

87. There are 24 pounds 8 ounces in a small bag of flour. If I buy 8 such bags, how many pounds do I get?

88. The circumference of a driving wheel is 16 feet. How many revolutions will it make in going a mile? (5,280 feet make a mile.)

89. The multiplicand is 21.9 and the multiplier is .37. What is the product?

90. The multiplier is 58 and the product is 45.82. What is the multiplicand?

91. The value of 6 horses and 68 cows is $3,870. If the value of each horse is $135, what is the value of each cow?

92. How many bushels of peanuts can be bought for $23.04, at $.08 a quart?

93. A dealer bought 9 tons of hay at $12½ a *ton, and sold* it for $145⅛. How much did he gain?

94. James lacks $1.75 of having $20. How much money has he? How much must he get to have $25?

95. A merchant sold 111 yards of silk, in equal parts, to 6 ladies. How much did each buy?

96. How many square yards in a lawn 36.5 yards long and 28.25 yards wide?

97. The difference between two numbers is 2,685. The smaller number is 3,168. What is the larger number?

98. A man sold land at $45 an acre, receiving $7,200 for it. How much would he have received for it if he had sold it for $56 an acre?

99. Henry earns $36 a month and his father earns $24 more. How many dollars do both together earn in one year?

100. A man paid \$38$\frac{3}{5}$ for a set of harness, and 14 times as much for a span of horses. How much did he pay for both?

101. A boy bought 3 bushels of peanuts at $1.95 a bushel, and sold them at $.04 a pint. How much did he gain?

102. If 9 pounds of tea cost $5.85, at the same price per pound how many pounds can be bought for $30.55?

103. How many pounds are there in 89.6 ounces avoirdupois?

104. How many cubic feet of air are there in a room 14 feet long, 11 feet wide, and 9 feet high?

105. If a grain dealer bought 238 bushels of corn at $.42 a bushel, for how much must he sell it per bushel to gain $21.42?

106. A man's salary is $1,400 a year, and he spends ⅝ of it. How much does he save? How many eighths of it does he save?

107. A grocer bought butter at $.24 a pound, paying $18.24 for it, and sold it at $.32 a pound. How much does he gain?

108. A man earns $46⅓ a month and his son earns ⅓ as much. How much do they both earn in one month?

109. A farmer took to a store 16 pounds of butter and 48 dozens of eggs. If he is paid $.32 a pound for the butter and $.18 a dozen for the eggs, how many yards of muslin at $.16 a yard can he receive in exchange?

110. The Civil War began in 1861. How many years ago was that?

111. A tub of butter weighs, with the tub, 42¾ pounds. The tub weighs 8½ pounds. What is the butter worth at 24 cents a pound?

112. What is one ninth of 26 days 2 hours 48 minutes 45 seconds?

A SUMMARY

OF

DEFINITIONS, PRINCIPLES, AND RULES.

A number is one or a collection of ones of the same kind.

Figures are merely the symbols of numbers.

We add, subtract, multiply, divide, and compare numbers, not figures.

Operations with figures always *represent* operations that may be performed with objects.

The study of numbers should always begin with the use of objects. But the use of objects should be discontinued as soon as the pupil has learned by their use how to regard numbers rightly, independent of objects.

Counting is the fundamental operation of Arithmetic.

As soon as the number ten is reached, the pupil should be taught to group numbers by tens.

Figures have two values—a shape-value, or *simple* value, and a place-value, or *local* value.

The same figure represents the same number, always and everywhere. But the kind of number it represents is determined by some sign or word, or by its place with reference to the decimal point.

The decimal point is merely a point from which to count the places of figures.

The zero has only one use, viz., to fill a place that would otherwise be vacant between some figure and the decimal point.

Moving a figure one place toward the left makes its value ten times what it was before.

When ten units of one kind invariably make one of the next higher, the number is simple.

Decimal fractions, numbers in Federal Money and in the Metric system are all simple numbers.

When some number of units other than ten of one kind make one of the next higher, the numbers are compound; mixed numbers, and generally numbers in the denominate tables, are compound.

Addition, Subtraction, Multiplication, and Division are the same for simple and for compound numbers; and the operations are performed in essentially the same way, varied only as the varying ratio of increase makes necessary.

Addition is finding a number that shall contain as many units as several other numbers taken together.

Subtraction is finding how many more units in one number than in another.

Addition, Subtraction, or Comparison can be performed with numbers of the same kind only.

RULE FOR WRITTEN ADDITION. Represent numbers of the same kind in vertical columns. Begin with the lowest denomination. From the *sum of* this denomination carry one to the next higher for each number of this kind that will make one of the next higher. So continue till the work is done.

RULE FOR WRITTEN SUBTRACTION. Write the figures for the smaller number under figures standing for units of the same kind in the larger number. Begin with the lowest denomination and take each number in the subtrahend from the like number in the minuend. If in any case a number in the subtrahend exceeds its like in the minuend, take one of the next higher kind in the minuend, change it to the kind required, add its value to the number of the lower kind in the minuend, and subtract from their sum.

In addition and subtraction, *remembered results from the table* must be used, else the work changes to mere counting.

Multiplication is taking a number as many times as there are units in another, to find the result.

The product is always of the same kind as the multiplicand.

The multiplier always means "times."

RULE FOR WRITTEN MULTIPLICATION. Begin with the lowest denomination and multiply each denomination of the multiplicand by the multiplier, "carrying" exactly as in addition. If there is more than one denomination in the multiplier, use each as directed, but multiply products by ten, one hundred, etc., by writing the figures one, two, or more places to the left, as the case may be. Add the partial products.

Division is finding how many times one number in another, or it is finding one of the equal parts of a number. In the first case the dividend and divisor must be of the same kind, and the quotient means "times." In the second case the dividend and quotient must be alike.

Rule for Division. Begin with the highest denomination of the dividend, find how many times the divisor is contained in it, or find the part of it which the divisor indicates; multiply together the divisor and the quotient thus found, and take the product from that part of the dividend that was used. If no quotient can be got from the highest denomination of the dividend, change that denomination to the next lower and add it to the given number of the next lower, and then proceed to divide. If there is a remainder after the subtraction of any product, use it in the same way.

No multiplication or division is possible except by using *remembered products or quotients* from the multiplication table.

Division is the opposite of multiplication; the dividend corresponds to the product—the divisor and quotient to its factors. When the divisor corresponds to the multiplicand the quotient is "times"; when it corresponds to the multiplier the quotient is like the dividend.

All these definitions, principles, and rules we have learned from our work in this book. They are placed together here for convenience and review, but need not be committed to memory.

ANSWERS.

Lesson 61.—(4) 234 cts. (5) 269 yds. (6) 310 qts. (7) 728 mls. (8) 789 sheep. (9) 1,077 bbls. (10) 1,504 bu. (11) 518 bu. (12) 1,381 acres. (13) 8,767 lbs. (14) $817.62. (15) $84.10. (16) $35.95. (17) 3.405 tons. (18) $80.

(1) 1,881. (2) 9.54. (3) 1,856. (4) 315.61. (5) 29.73. (6) 1.97.

(1) 238. (2) 226. (3) 346. (4) 277. (5) 322. (6) 2,547. (7) 2,284. (8) 1,996. (9) 2,017. (10) $88.81. (11) $83.356. (12) 83.569.

Lesson 62.—(5) 165 sheep. (6) $72. (7) 147 ft. (8) 70 yrs. (9) 272 bbls. (10) 136 words. (11) 1,876 men. (13) $974. (14) 280 gals. (16) 150 bricks. (17) 79 mls. (18) $835. (19) $105.85. (20) $670.67. (21) $280.78. (22) $508.85. (23) $415.77.

(1) 45. (2) 32. (3) 35. (4) 24. (5) 54. (6) 73. (7) 54. (8) 55. (9) 60. (10) 45. (11) 28. (12) 28. (13) 37. (14) 29. (15) 36. (16) 47. (17) 35. (18) 48. (19) 47. (20) 39. (21) 289. (22) 359. (23) 169. (24) 416. (25) 221. (26) 276. (27) 707. (28) 85. (29) 209. (30) 838. (31) 148. (32) 758. (33) 256. (34) 677. (35) 578. (36) 506. (37) $18.58. (38) $4.50. (39) $6.25. (40) $3.65. (41) 87.50. (42) $4.75. (43) $6.875.

Lesson 63.—(7) $71\frac{5}{8}$ lbs. (9) $20\frac{14}{16}$ lbs. (16) $$1\frac{3}{4}$. (17) 39 yds. (18) $$99\frac{3}{4}$. (19) 2 lbs. (20) $4\frac{3}{5}$ acres. (21) Walter's age, $9\frac{3}{8}$ yrs., and 5 yrs. older than Nellie.

Lesson 64.—(1) 8 gals. 1 qt. (2) 21 yds. (3) 15 bu. 1 pk.; 2 bu. 1 pk. (4) 179 lbs. 2 oz.; 3 lbs. 14 oz. (5) 17 gals. 3 qts. 1 pt. (6) 2 ft. 8 in. (7) 5 bu. 2 pks. 4 qts.; 6 qts. (8) 8 lbs.; 12 oz. (9) 10 gals. 3 qts. 1 pt. (10) 8 yds. 6 in. (11) 12 bu. 2 pks. (12) 4 bu. 2 pks. 4 qts. (13) 2 ft. 8 in. (14) 3 lbs. 9 oz.

Lesson 65.—(1) 3,820 ft. (2) 1,328 men. (3) 158 bu. (4) 119.375 acres. (5) $$49\frac{3}{8}$. (6) $46\frac{10}{12}$ ft. (7) 6 lbs. 4 oz. (8) 15 yds. 1 ft. 9 in. (9) $429. (10) 45 lbs. 4 oz. (11) $4,331.91. (12) $55\frac{5}{8}$ mls. (13) 215.64 mls. (14) 227 bu. 3 pks.; 7 bu. (15) $164\frac{5}{8}$ lbs.; $3\frac{7}{8}$ lbs. (16) $$94\frac{1}{8}$. (17) $1,643.15. (18) $68.65.

(1) $102\frac{3}{4}$. (2) $63\frac{3}{8}$. (3) $127\frac{6}{7}$. (4) 157. (5) $130\frac{1}{9}$. (6) 21 bu. 1 pk. 3 qts. (7) 23 yds. 11 in. (8) 25 lbs. (9) 26 gals. (10) $34\frac{4}{7}$. (11) $28\frac{5}{8}$. (12) $31\frac{2}{5}$. (13) $2\frac{8}{13}$. (14) $10\frac{7}{8}$. (15) $1\frac{4}{5}$. (16) $7\frac{3}{4}$. (17) $12\frac{1}{8}$. (18) $3\frac{3}{4}$. (19) $7\frac{1}{5}$. (20) 3 lbs. 12 oz. (21) 4 gals. 3 qts. (22) 5 yds. 2 ft. 8 in. (23) 7 bu. 2 pks. 7 qts. 1 pt.

Lesson 66.—(8) 672 cts. (9) 325 sheep. (10) 468 mls.; 546 mls. (11) 380 bu. (12) $292. (13) 243 cu. ft. (14) 384 sq. in. (15) 455 cts. (16) 360 yds. (17) 504 lbs. (18) $7.65. (19) 168 hrs.

(1) 276 sheep. (2) $584. (3) 612 books. (4) 261 posts. (5) 553 birds. (6) 306 desks. (7) 432 pens. (8) 380 boys. (9) 588 men. (10) 266 pinks.

Lesson 67.—(1) 896 cts. (2) 297 ft. (3) 113 gals. 1 qt. (4) $1,824. (5) $4\frac{3}{8}$ pks. (6) $265.05. (7) 689 lbs. (8) 4,032 sq. in. (9) 2,546 sq. ft. (10) 212\frac{4}{5}$. (11) $63.70. (12) $16.20. (13) 8,640 cu. in. (14) 780\frac{3}{4}$. (15) $19.44. (16) 1,155 cu. in. (17) $64.60. (18) 840 mls. (19) 7 melons. (20) $30\frac{4}{5}$ mls. (21) 82 bu. 2 pks. (22) $280.49. (23) $5.92 gain. (24) 52\frac{1}{5}$. (25) $6.24. (26) $3,145.

Lesson 68.—(1) 8,000 rods. (2) 8,100 steps. (3) $153. (4) 258 cts. (5) $13.36. (6) $10.40. (7) 22 cts. (8) $6.93. (9) 20 bu. 1 pk. 3 qts. (10) 4,836. (11) 48\frac{1}{8}$. (12) 64 cts. (13) 1,440 cu. in. (14) 28 lbs. 14 oz. (15) 542\frac{2}{5}$. (16) 89 yds. 1 ft. (17) $156.60.

(1) 1,274. (2) 5,742. (3) $667.55. (4) 153.28 ft. (5) 205.61 bu. (6) 3005.1 gals. (7) 105 lbs. (8) 25 bu. 3 pks. 2 qts. (9) 82 lbs. 2 oz. (10) 337 yds. (11) 1,260. (12) $25.02. (13) 22.032. (14) 112.48.

Lesson 69.—(12) 247 hats. (13) $48. (14) 94 bbls. (15) $165. (16) 227 gals. (17) $75. (18) 97 papers. (19) 48 hrs. (20) $465. (21) 68 cts. (22) 139 wks. (23) 158 cans. (24) 45 hrs. (25) 216 pks. (26) 176 bu. (27) $.85. (28) 218 sq. yds. (29) 593 lines.

(1) 142. (2) 197. (3) 135. (4) 138. (5) 213. (6) 266. (7) 63. (8) 57. (9) 144. (10) 107.

Lesson 70.—(8) $5\frac{3}{8}$ yds. (9) $4\frac{2}{3}$ mls. (10) $6\frac{1}{3}$ times. (11) 6\frac{5}{8}$. (12) 257 lbs. (13) $14.65. (14) 88 bu. 2 pks. (15) 148 qts. (16) 46\frac{2}{3}$. (17) 2 lbs. 9 oz. (18) 6 yds. 1 ft. 6 in. (19) 14\frac{3}{4}$.

(1) $93.24. (2) $38.45. (3) $67.83. (4) $84.53. (5) $94\frac{5}{7}$. (6) $86\frac{2}{3}$. (7) $74\frac{5}{8}$. (8) $67\frac{5}{6}$. (9) 8 yds. 2 ft. 9 in. (10) 75 gals. 3 qts. 1 pt. (11) 38 bu. 1 pk. 7 qts. (12) 27 lbs. 13 oz.

Lesson 71.—(1) 123 doz. (2) $38. (3) 136 lbs. (4) $.65. (5) $165. (6) 36 mos. (7) 48 lbs. (8) $74. (9) 85 bu. (10) 96 trees. (11) 109 cu. yds. (12) 66 yds. (13) $4.85. (14) 68 yrs. (15) 95 bu. (16) 27 days. (17) 97 hrs. (18) 81 acres. (19) 115 mls. (20) 83 companies. (21) 37 sq. ft. (22) 48 bbls. (23) 36 gals. (24) 14 yrs. (25) $9.

(1) 247. (2) 234. (3) 21. (4) 37. (5) 93. (6) 38. (7) 29. (8) 29. (9) 15. (10) .027. (11) .046. (12) .038.

Lesson 72.—(1) $229.10; $37.525. (2) 25 yds.; 12.5 yds.; 3.75 yds. (3) 472.5 gals.; 305.55 gals.; 114.975 gals.; 38.304 gals. (4) 27 rods; 8.3 rods. (5) $4.025; $7.425. (6) 7 bu.; 19 bu. (7) 344.5 mls.; 123.225 mls.; 28.4875 mls. (8) 15 hrs.; 1.7 hrs. (9) 11.9 tons; 142.8 tons; 61.404 tons. (10) 569.25 cu. ft.; 165.825 cu. ft.; 228.69 cu. ft. (11) 31 perches; 2.5 perches; .9 of a perch.

(1) 32.3. (2) 2.088. (3) .378. (4) 17.86. (5) .576. (6) .686. (7) .057. (8) 8. (9) 29. (10) 4.6. (11) .09. (12) .45. (13) 19. *(14) 25.*

Promiscuous Problems.

WRITTEN EXERCISES.

(1) 67 yrs. (2) $202. (3) 576 oz. (4) 274 sheep. (5) $\$4\frac{1}{2}$. (6) $83.25. (7) $74\frac{7}{8}$ bu. (8) $2,394. (9) 103 mls. (10) $\$1,591\frac{1}{5}$. (11) 756 cu. ft. (12) $66.85. (13) $\$9\frac{3}{10}$. (14) $3,162.18. (15) 351 lbs. 1 oz.; 5 lbs. 14 oz. (16) $104.50. (17) $\$2.05\frac{1}{8}$. (18) 9 bu. (19) 2 wks. 6 days 14 hrs. 48 min. 25 sec. (20) $9. (21) 2,058 bricks. (22) $38\frac{3}{5}$ cords. (23) 23 sq. yds. 8 sq. ft. (24) $660. (25) $50. (26) $13.32. (27) 110 acres. (28) 3,888 sq. in. (29) 38 yds. (30) 252 sq. ft. (31) $31.20. (32) 9.85 tons. (33) $2,048. (34) $2.25. (35) $2,075. (36) 38.6 bu. (37) $833.63. (38) 64 cu. in. (39) 84 qts. (40) 59 wks. (41) 18 days. (42) $43.35. (43) 4,254 persons. (44) $15.20. (45) $146\frac{7}{8}$ mls. (46) 25 yds. (47) $3.15. (48) 8,861 sq.in. (49) $28.80. (50) $825. (51) $\$106\frac{3}{16}$. (52) 648 cu. in. (53) $\$5\frac{5}{8}$. (54) 208 wks. 4 yrs. (55) 150 doz. (56) $9.12. (57) $565.58. (58) 235 (59) 3 yds. (60) 1 gal. 2 qts. 1 pt. (61) $13.50. (62) $99. (63) 455 lbs. (65) $1,760; 55 acres. (66) 4,464 mls. (67) 30 bu. (68) 125 bu. (69) $68\frac{5}{8}$ mls. (70) 17.63 cu yds. (71) $632. (72) $3,264.70. (73) $38.10. (74) 1 day 12 hrs. 30 min. (75) $318.95. (76) 258 bu. (77) $\$57\frac{5}{8}$. (78) 98 lbs. (79) 8 yds. 1 ft. 9 in. (80) .83. (81) $.95. (82) 10,080 min. (83) $\$27\frac{1}{3}$. (84) 2 pks. 4 qts. (85) 560 cu. in.; 2 gals. and 98 cu. in. over. (86) $30.09. (87) 196 lbs. (88) 330 revolutions. (89) 8.103. (90) .79. (91) $45. (92) 9 bu. (93) $\$32\frac{5}{8}$. (94) $18.25; $6.75. (95) 18 yds. 1 ft. 6 in. (96) 1,031.125 sq. yds. (97) 5,853. (98) $8,960. (99) $1,152. (100) $579. (101) $1.83. (102) 47 lbs. (103) 5.6 lbs. (104) 1,386 cu. ft. (105) $.51. (106) $525. (107) $6.08. (108) $\$61\frac{3}{5}$. (109) 86 yds. (111) $8.22. (112) 2 days 21 hrs. 38 min. 45 sec.

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